

# Networks and hypernetworks 3

Networks in economy

Rui Vilela Mendes

<http://label2.ist.utl.pt/vilela/>

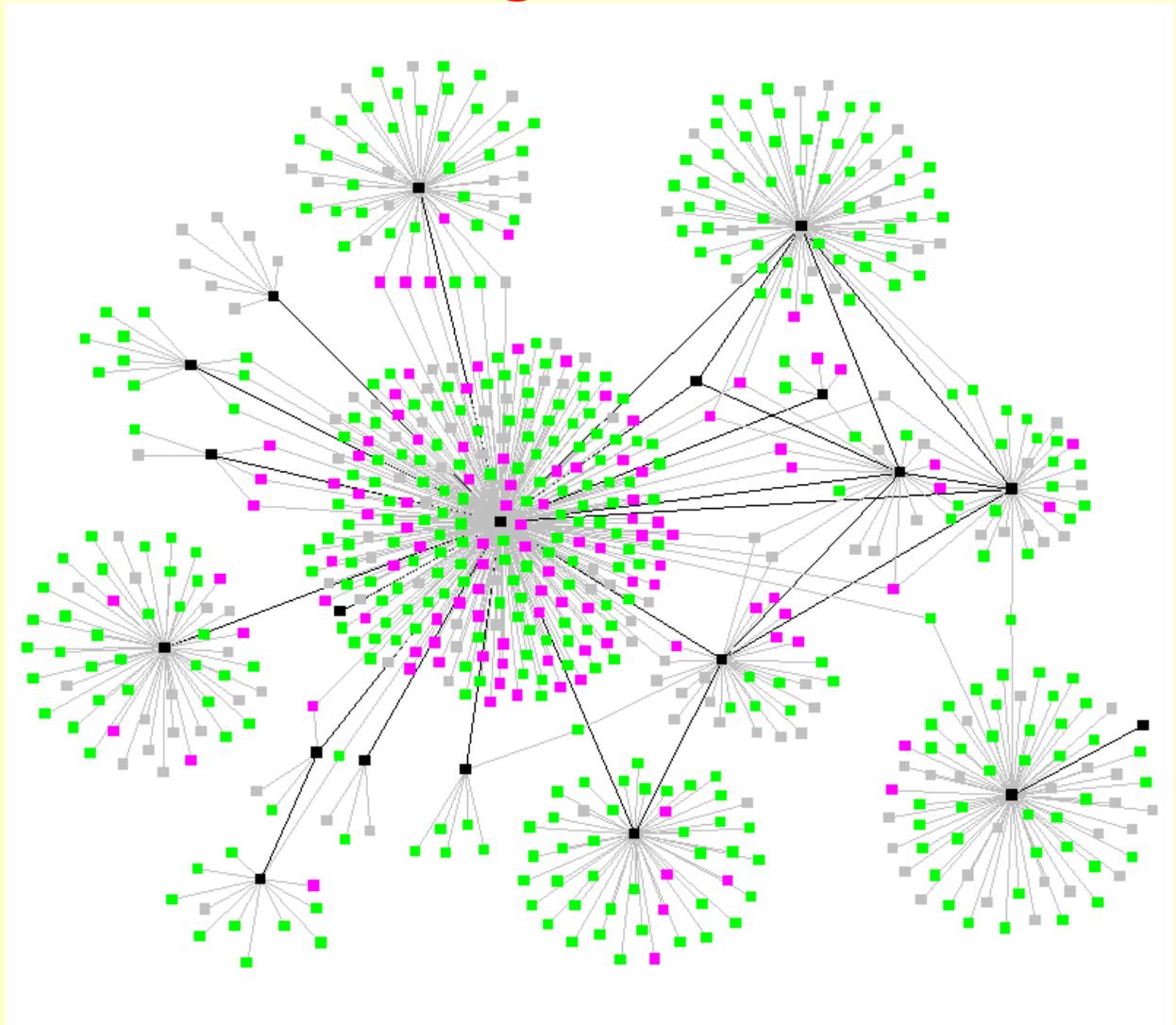




# Viral marketing

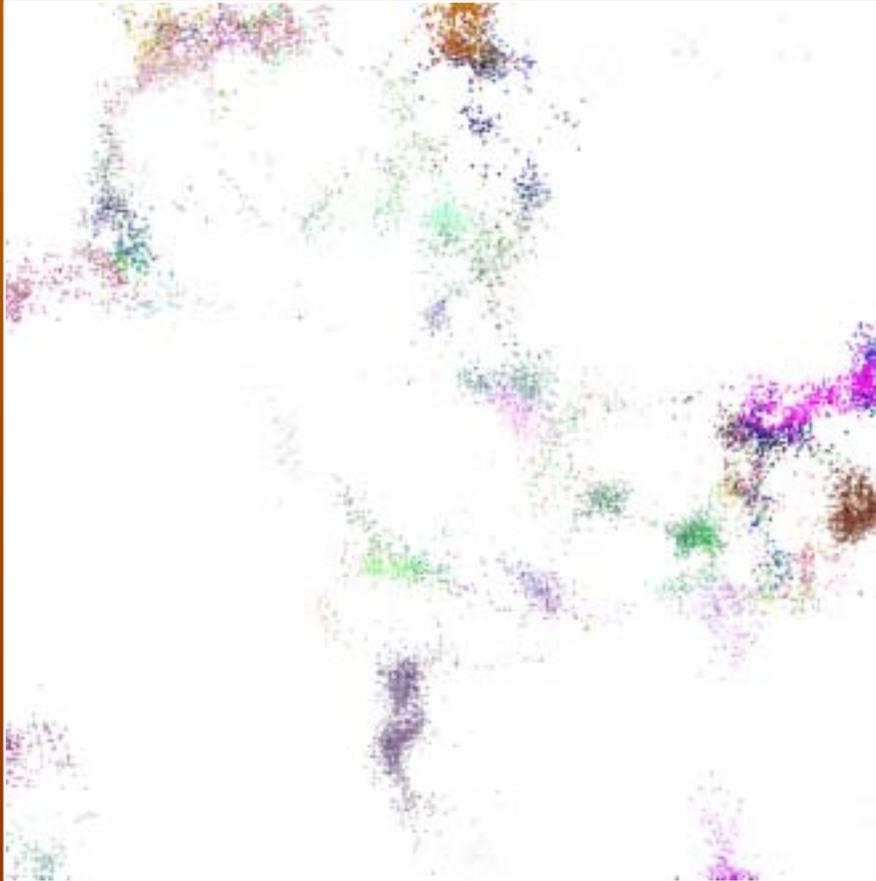
**Hubs:**

**‘broadcast’ weakly  
infectious viruses,  
ideas**



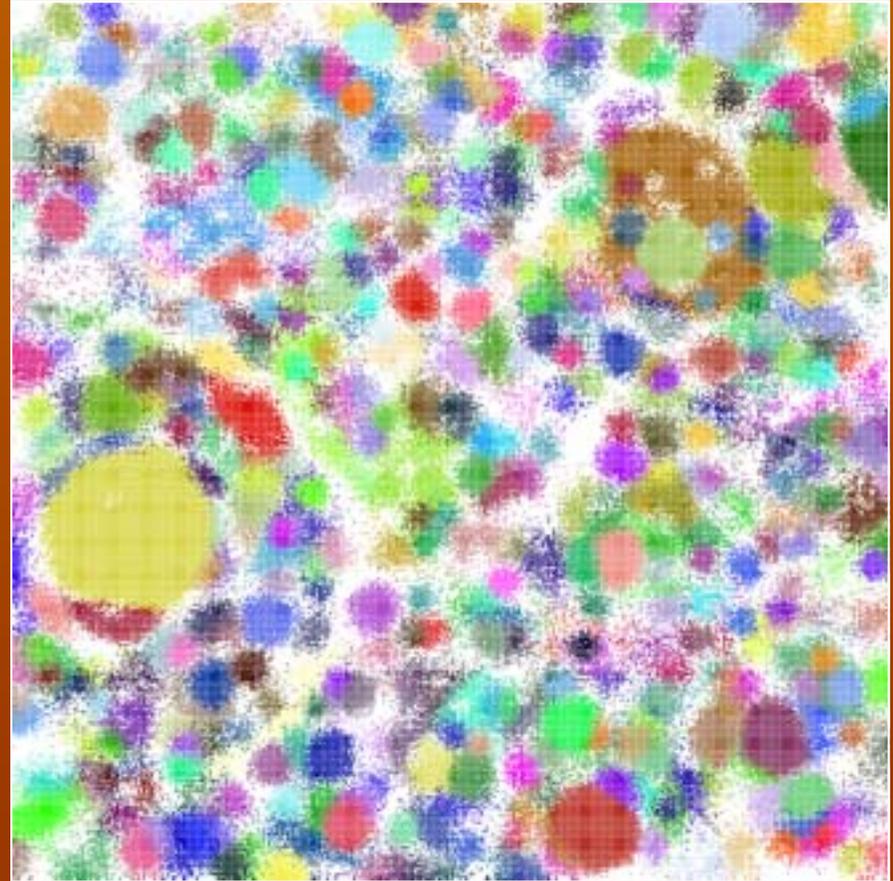
**black:** opinion leaders  
**red:** influenced  
**green:** uninfluenced  
**grey:** undecided

500 randomly chosen users



Day 

500 *most active* users



Day 

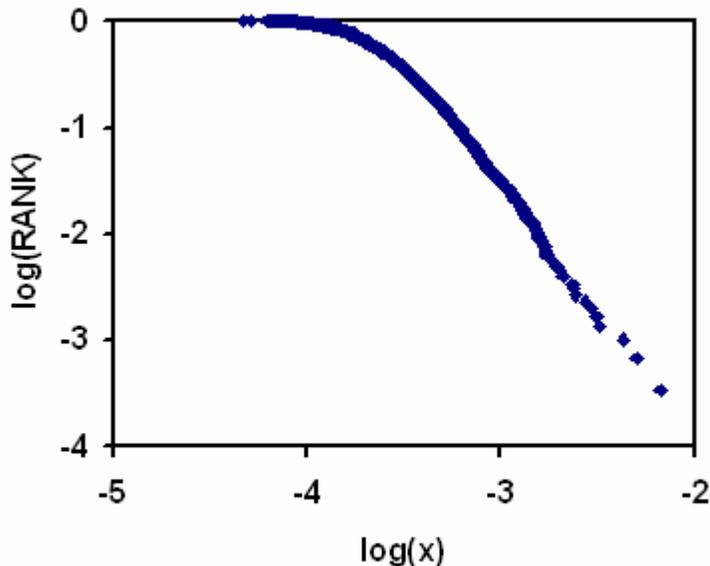
# Empirical wealth distributions

Two typical forms:

Large wealth: **Pareto's law**  
(*power-law distribution*):

$$P(w) \propto w^{-1-\alpha}$$

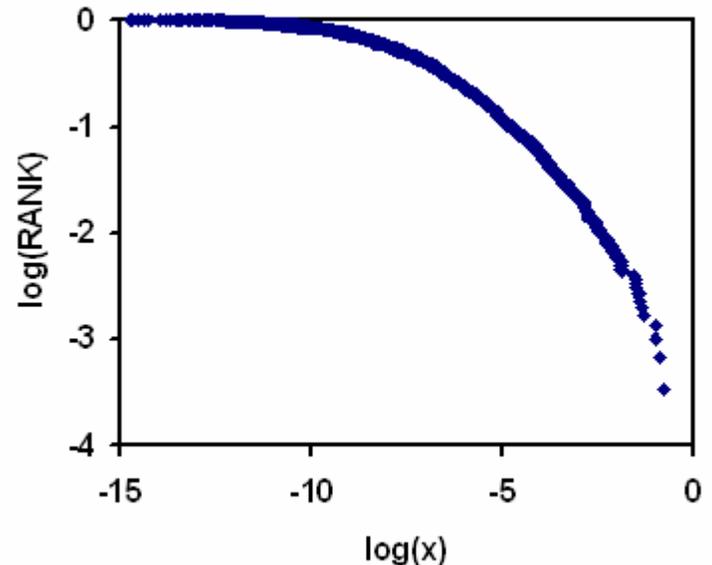
**Pareto's (power-law) distribution**



Small wealth: **Gibrat's law**  
(*log-normal distribution*):

$$P(w) = \frac{1}{w\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2} \log^2 \frac{w}{w_0}\right]$$

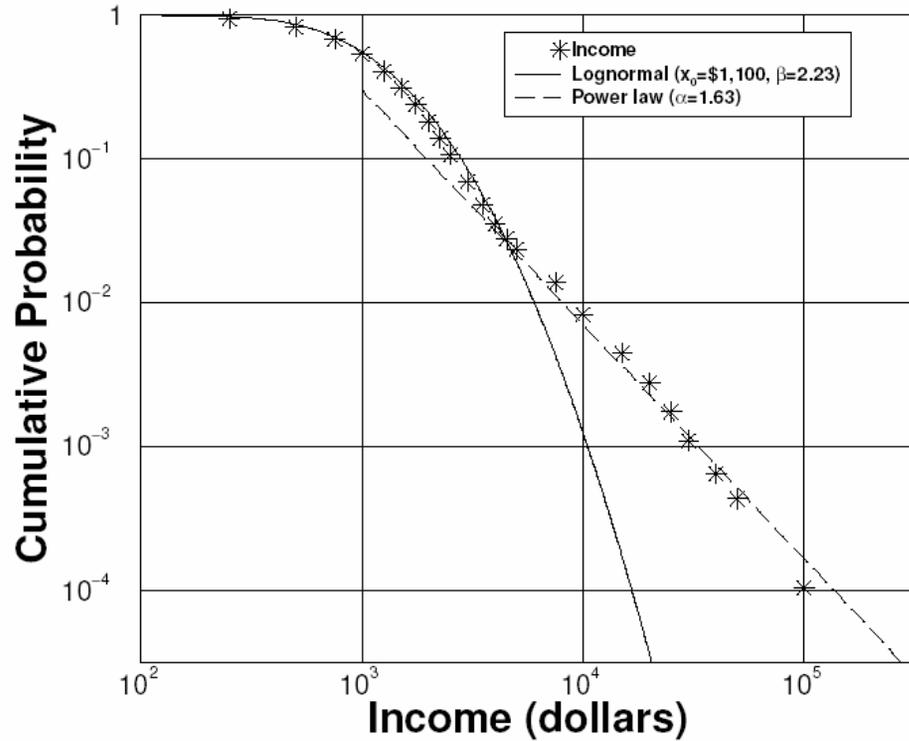
**Gibrat's (log-normal) distribution**



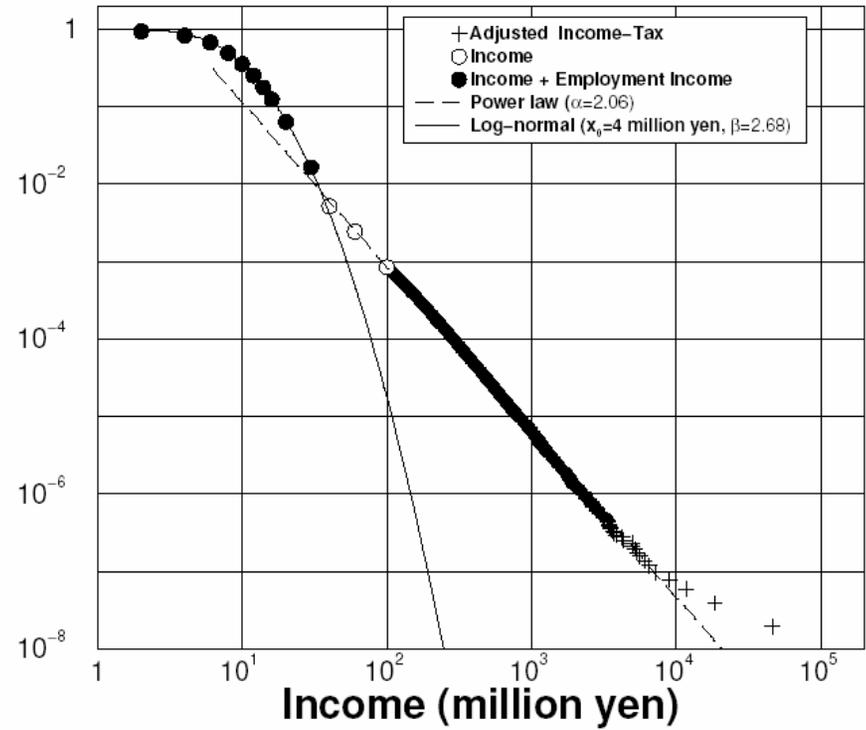
Cumulative distribution:  $P_{>}(x) = \int_x^{\infty} P(x') dx'$      $x \equiv w/w_{tot}$

# Personal Income Distribution

## U.S.A. 1935-36 (\$)

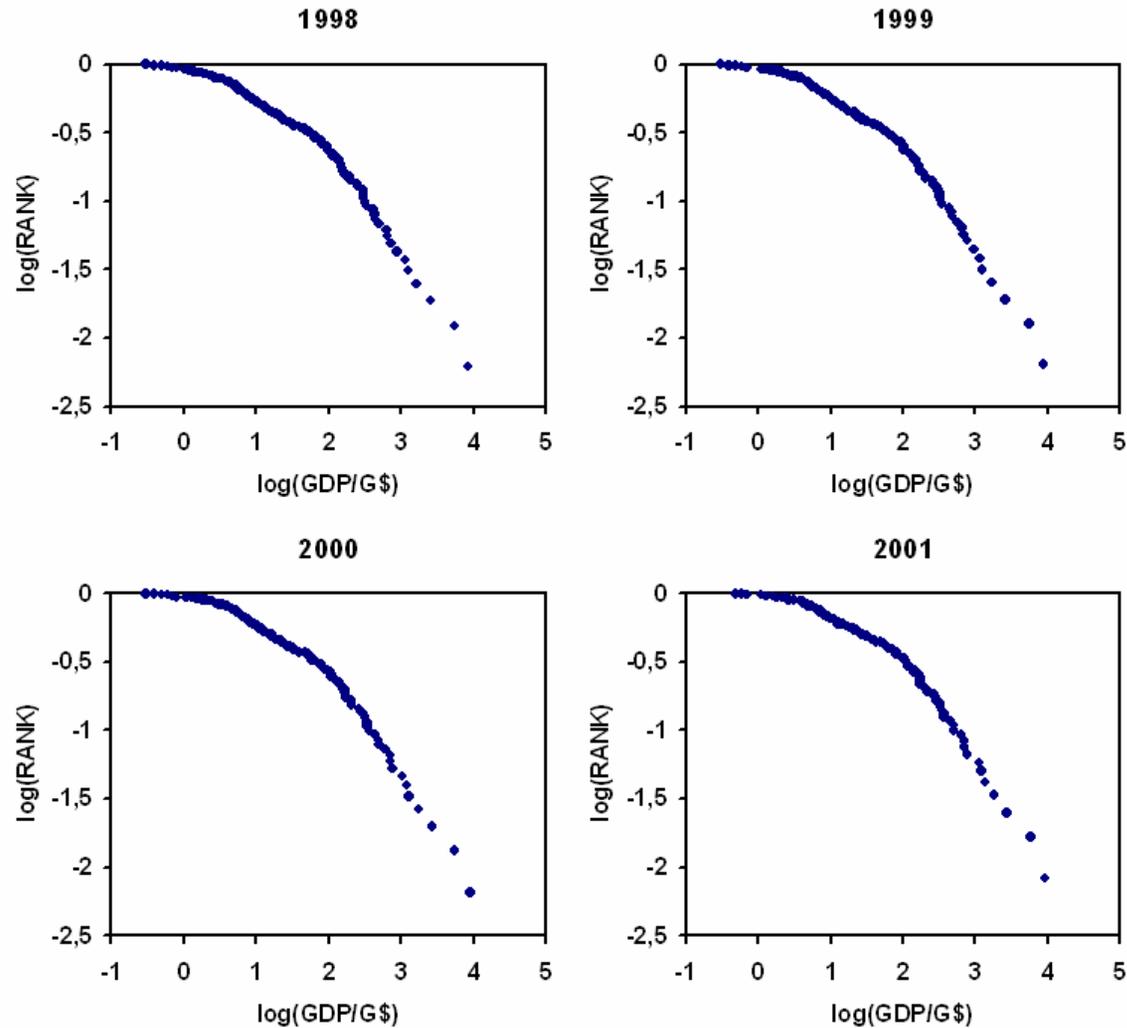


## Japan 1998 (M¥)



Log-normal distribution with power-law tails (mixed form)

# Gross Domestic Product Distribution (*GDP*)



All countries; 1998, 1999, 2000, 2001 (**G\$**): log-normal and power-law (mixed)

# Empirical forms of “wealth” distributions:

The most general form of  $P(w)$  is  
“mixed”:

Combination of a power-law and a  
log-normal distribution



Theoretical models that can reproduce the mixed form

# Independent agents models

Purely multiplicative stochastic process:

$$\dot{w}_i(t) = \eta_i(t)w_i(t)$$

$w_i(t)$  = wealth of agent  $i$  at time  $t$

$\eta_i(t)$  = Gaussian process (mean  $m$  and variance  $2\sigma^2$ )

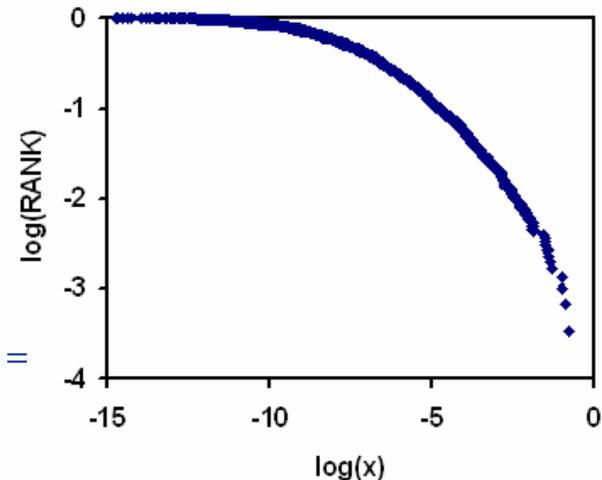


Log-normal distribution

$$w_i(t+1) = w_i(t) + \eta_i(t)w_i(t) = [1 + \eta_i(t)]w_i(t)$$

$$\begin{aligned} \log[w_i(t+1)] &= \\ &= \log[w_i(t)] + \log[1 + \eta_i(t)] = \\ &= \log[w_i(t-1)] + \log[1 + \eta_i(t-1)] + \log[1 + \eta_i(t)] = \\ &= \dots \end{aligned}$$

Gibrat's (log-normal) distribution



# Independent agents models

Multiplicative stochastic process with a lower boundary:

$$\dot{w}_i(t) = \eta_i(t)w_i(t) \quad w_i(t) > w_{min}$$

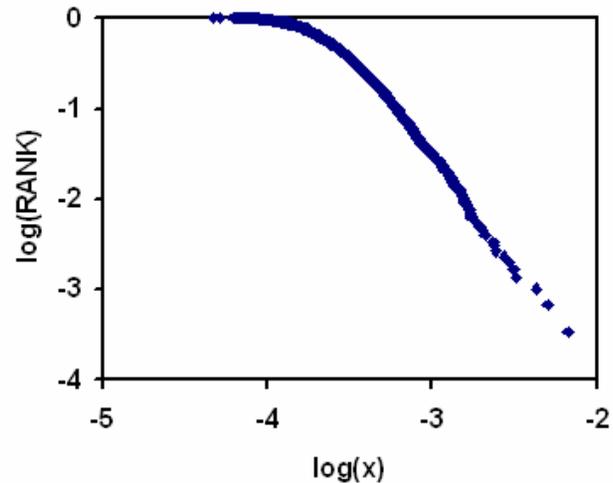
Multiplicative-additive stochastic process:

$$\dot{w}_i(t) = \eta_i(t)w_i(t) + \xi_i(t) \quad \langle \log \eta_i(t) \rangle < 0$$



Power-law distribution

Pareto's (power-law) distribution



# Model of Bouchaud and Mézard (*BM*)

Interactive multiplicative stochastic process:  
wealth evolution is determined by the interactions among economic agents

Wealth evolution with  $N$  agents:

$$\dot{w}_i(t) = \eta_i(t)w_i(t) + \sum_{j \neq i} J_{ij} w_j(t) - \sum_{j \neq i} J_{ji} w_i(t)$$

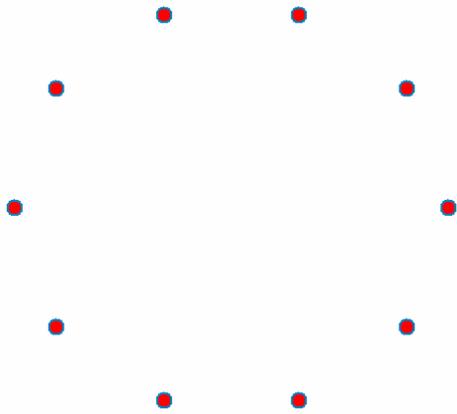
$w_i(t)$  = wealth of agent  $i$  at time  $t$

$\eta_i(t)$  = Gaussian process (mean  $m$  and variance  $2\sigma^2$ )

$J_{ij}$  = fraction of wealth flowing from  $j$  to  $i$

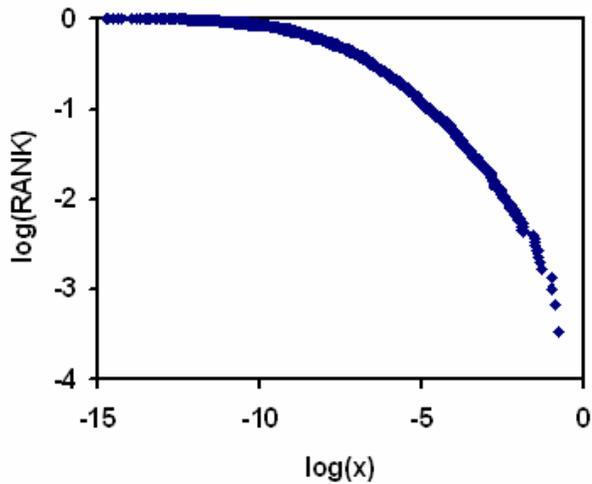
# Independent agents

$$J_{ij} = 0 \quad \forall i, j$$



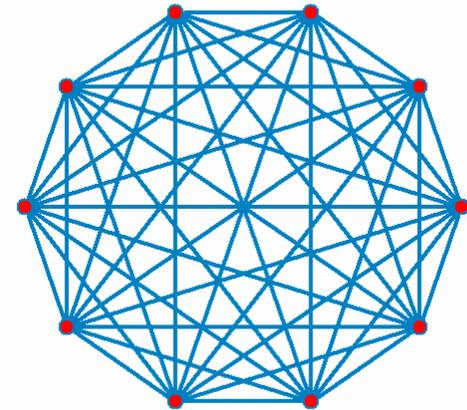
$$\dot{w}_i(t) = \eta_i(t) w_i(t)$$

Gibrat's (log-normal) distribution



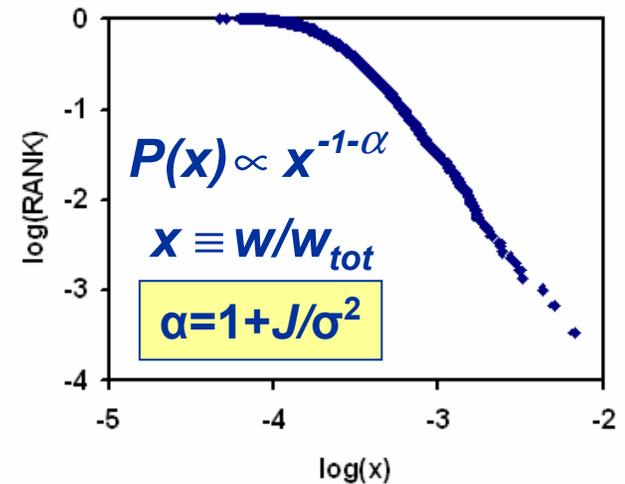
# Mean field

$$J_{ij} = J/N \quad \forall i, j$$



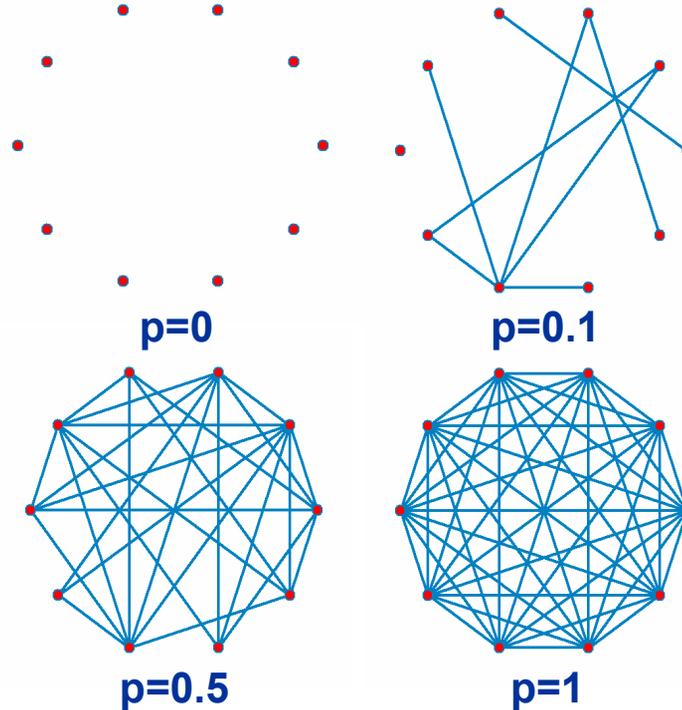
$$\dot{w}_i(t) = \eta_i(t) w_i(t) + J \bar{w}(t) - J w_i(t)$$

Pareto's (power-law) distribution



# *BM* model on random graphs

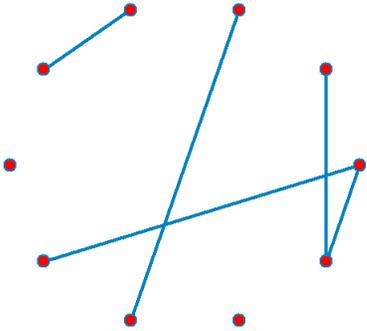
- Start with a set of  $N$  isolated vertices;
- For each pair of vertices draw a link with uniform probability  $p$ .



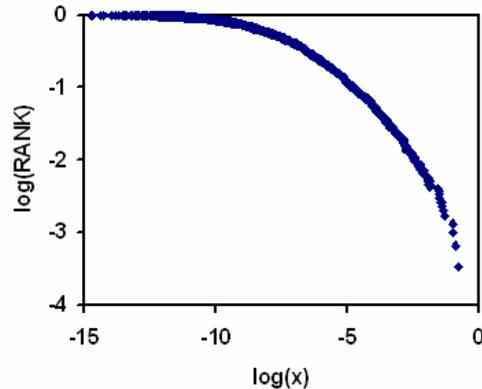
# BM model on random graphs

The wealth distribution  $P(w)$  changes suddenly from log-normal ( $p < p_c$ ) to power-law ( $p > p_c$ )

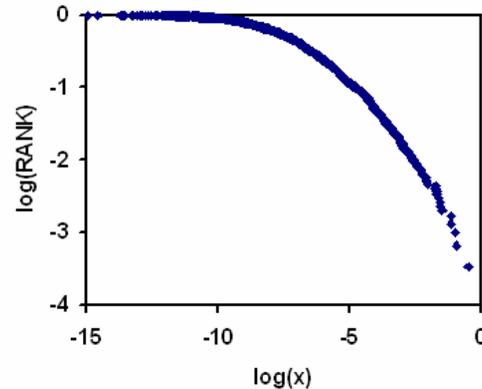
$$p = N^{-1.5}$$



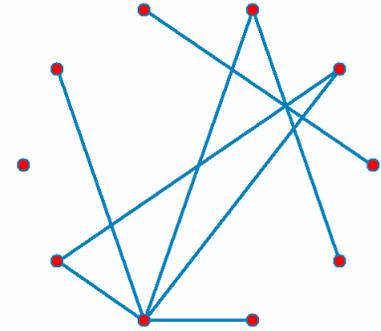
Random ( $p = N^{-1.5}$ )



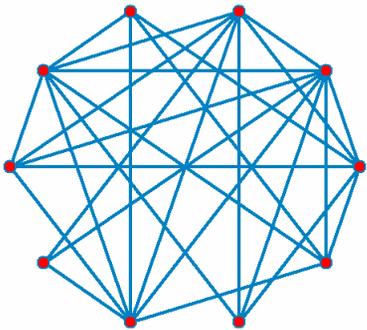
Random ( $p = N^{-1}$ )



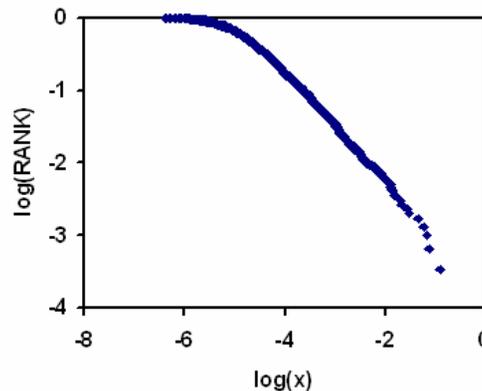
$$p = N^{-1}$$



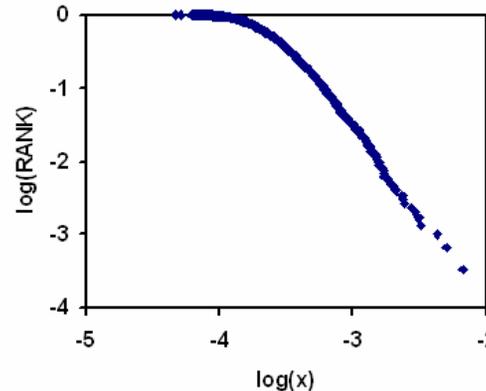
$$p = N^{-0.5}$$



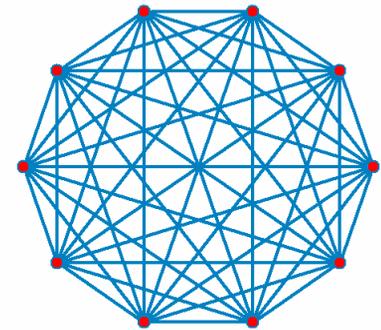
Random ( $p = N^{-0.5}$ )



Random ( $p = 1$ )



$$p = N^0 = 1$$



Simulation parameters:  $N=3000$   $T=10000$

$$J=0.05 \quad \langle \eta \rangle = 1 \quad \langle \eta^2 \rangle - \langle \eta \rangle^2 = 0.1$$





# The networks of the corporate elite

- Individuals, at the core of the network, control the diffusion of information in the network
- Corporate governance practices spread through shared directors
- Firms are more likely to adopt an acquisition strategy if they share a director with a company that has an acquisition strategy
- Anti-takeover strategies diffuse along director networks

# The global corporate elite

- Network of overlapping membership among directors of the world's (500) leading corporations and transnational policy boards
- 500 leading corporations
- 7 global policy groups
- 4 transnational business councils

*(W. K. Carroll and J. P. Sapinsky, International Sociology 25 (2010) 501-538)*

# The global corporate elite

## ■ Global policy groups

|  | No. of directors |      | Agenda priorities                              | Organizational form   | Core membership  | Geopolitical reach   |
|--|------------------|------|--|---|--|--|
|  | 1996             | 2006 |  |   |  |  |
| <i>1: Global policy groups</i>   |                  |      |  |   |  |  |
| International Chamber of Commerce<br>Est. 1919<br>Paris headquarters                 | 27               | 25   | Corporate self-regulated, global laissez faire | International business organization; government lobbyist; linking to locals (national committees) | 7000 corporations from 130 countries   | Global; corporations and regional committees worldwide, including the Americas, Europe, the Middle East, Africa and the Asia-Pacific |
| Bilderberg Conferences<br>Est. 1952<br>Geneva origin; office in Leiden (Netherlands) | 112              | 135  | Economic order among 'heartland' states        | Secretive policy-planning and elite consensus-seeking forum                                       | 130 national and international corporate, govt, military and academic elite; no set membership | North Atlantic 'heartland'; draws elite representation from Western Europe and North America   |
| Trilateral Commission<br>Est. 1972<br>Washington, Paris & Tokyo headquarters         | 304              | 413  | Economic order among 'Triad' states            | Policy-planning and elite consensus-seeking forum; research task forces; discourse producer       | 350 national and international corporate, media, academia, public service and NGO elite        | 'The Triad'; draws elite representation from North Atlantic, Japan, ASEAN  |

# The global corporate elite

|  | No. of directors |      | Agenda priorities                          | Organizational form   | Core membership   | Geopolitical reach  |
|--|------------------|------|--|---|---|---|
|  | 1996             | 2006 |  |   |   |   |
| World Economic Forum<br>Est. 1971 (1987)<br>Geneva headquarters  | 55               | 47   | 'Global' economic order                    | Combined elite transnational business organization, and policy-planning and consensus-seeking forum; research task forces; discourse producer | 1000 top transnational corporations   | Global; draws elite representation from Western Europe, Central and Eastern Europe, Africa, North America, Latin America, Asia and Oceania                |
| International Advisory Board of the Council on Foreign Relations<br>Est. 1995<br>New York headquarters (CFR) | 35               | 33   | Strategic orientation of US foreign policy | <i>CFR</i> : policy-planning and consensus-seeking forum, discourse producer, task forces; <i>Advisory board</i> : Advisory function to CFR   | <i>CFR</i> : US citizens; 250 corporate members (US and foreign firms); <i>Advisory board</i> : 33 corporate and political elites from around the world | Global; Advisory board draws elite representation from Western Europe, Central and Eastern Europe, Africa, North America, Latin America, Asia and Oceania |
| World Business Council for Sustainable Development   | 116              | 185  | 'Global' environmental and economic reform | Combined elite transnational business organization, and policy-planning and   | 206 corporations (by invitation)  | Global; draws elite representation from Western Europe,   |

# The global corporate elite

## ■ Transnational business councils

|  | No. of directors |      | Agenda priorities   | Organizational form  | Core membership   | Geopolitical reach   |
|--|------------------|------|---|--|---|--|
|  | 1996             | 2006 |   |  |   |  |
| Est. 1995<br>Geneva headquarters   |                  |      |   | consensus-seeking forum; research task forces; discourse producer  |   | Central and Eastern Europe, Africa, North America, Latin America, Asia and Oceania   |
| UN Global Compact (Board)<br>Est. 2000<br>New York headquarters (UN)         | n/a              | 19   | Promotion of corporate social and environmental responsibility                  | Combined elite transnational organization; national or regional communication networks   | Board composed of representatives from business (12), labour (2), NGOs (4) and the UN (2)                       | Global; draws elite representation from Western Europe, Central and Eastern Europe, Africa, North America, Latin America, Asia and Oceania |
| <i>2: Transnational business councils</i>                                    |                  |      |   |  |   |  |
| <i>Europe</i>  |                  |      |   |  |   |  |
| European Round Table of Industrialists<br>Est. 1983<br>Brussels headquarters | 56               | 57   | Economic integration in Europe; European corporations' global economic position | Combined elite regional business organization, and policy-planning and consensus-seeking forum; lobbying; working groups; discourse producer | Corporate chief executives and chairpersons of major European-owned TNCs from industrial and technology sectors | European Union   |

# The global corporate elite

|   | No. of directors   |      | Agenda priorities   | Organizational form  | Core membership  | Geopolitical reach                 |
|---|--------------------|------|---|--|--|------------------------------------|
|   | 1996               | 2006 |   |  |  |                                    |
| <i>Europe and Asia</i>  |                    |      |   |  |  |                                    |
| EU–Japan Business Round Table<br>Est. 1995<br>Brussels & Tokyo headquarters                         | 26                 | 50   | Strengthening economic ties between EU and Japan  | Combined interregional business organization, and policy-planning and consensus-seeking forum; lobbying; task forces; discourse producer | 50 EU and Japanese (about half from each) corporate executives and directors                         | European Union and Japan           |
| <i>North Atlantic</i>   |                    |      |   |  |  |                                    |
| TransAtlantic Business Dialogue<br>Est. 1995<br>Washington headquarters                             | 68<br>(see note 3) | 33   | Economic integration and trade liberalization between the US and EU   | Combined elite interregional business organization, and policy-planning and consensus-seeking forum                                      | 31 US and EU CEOs/chairs   | United States and European Union   |
| <i>North America</i>  |                    |      |   |  |  |                                    |
| North American Competitiveness Council<br>Est. 2006<br>Ottawa, Mexico & Washington, DC headquarters | n/a                | 33   | Economic integration and trade liberalization in North America; North American corporations' global economic position | Combined elite regional business organization, and policy-planning and consensus-seeking forum   | 33 corporate executives total; 10 members nominated by each of Canada and Mexico, 13 nominated by US | North America (Canada, US, Mexico) |

# The global corporate elite

*Table 2 Policy-Board Memberships and Corporate Directorships, 1996 and 2006*

|   | A           | B          | (B-A)/A      |
|---|-------------|------------|--------------|
| Patterns of affiliation                             | 1996        | 2006       | % change     |
| <i>a</i> 1 corporate board                          | 7921        | 5248       | -33.7        |
| <i>b</i> 1 policy board                             | 419         | 650        | +55.1        |
| <b><i>c</i> 2+ corporate board</b>                  | <b>757</b>  | <b>611</b> | <b>-19.3</b> |
| <i>d</i> 2+ policy boards                           | 26          | 32         | +23.1        |
| <i>e</i> 1 corporate board and 1 policy board       | 109         | 138        | +26.6        |
| <i>f</i> 1 corporate board and 2+ policy boards     | 9           | 22         | +144.4       |
| <i>g</i> 2+ corporate boards and 1 policy board     | 72          | 57         | -20.8        |
| <i>h</i> 2+ corporate boards and 2+ policy boards   | 27          | 27         | 0            |
| <b>Total: members of the corporate-policy elite</b> | <b>1000</b> | <b>887</b> | <b>-11.3</b> |
| Grand total   | 9330        | 6785       | -27.3        |

# The global corporate elite

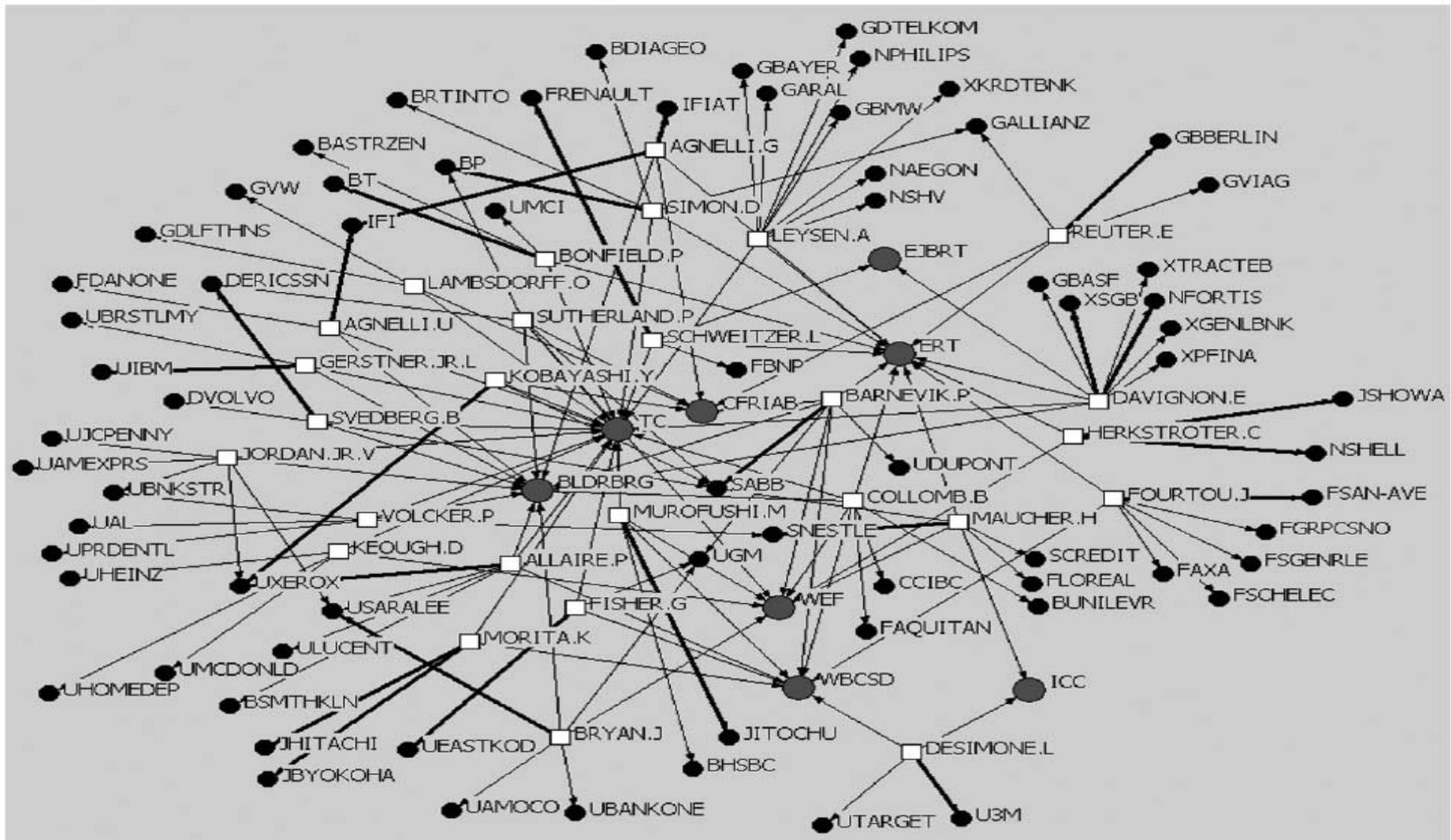
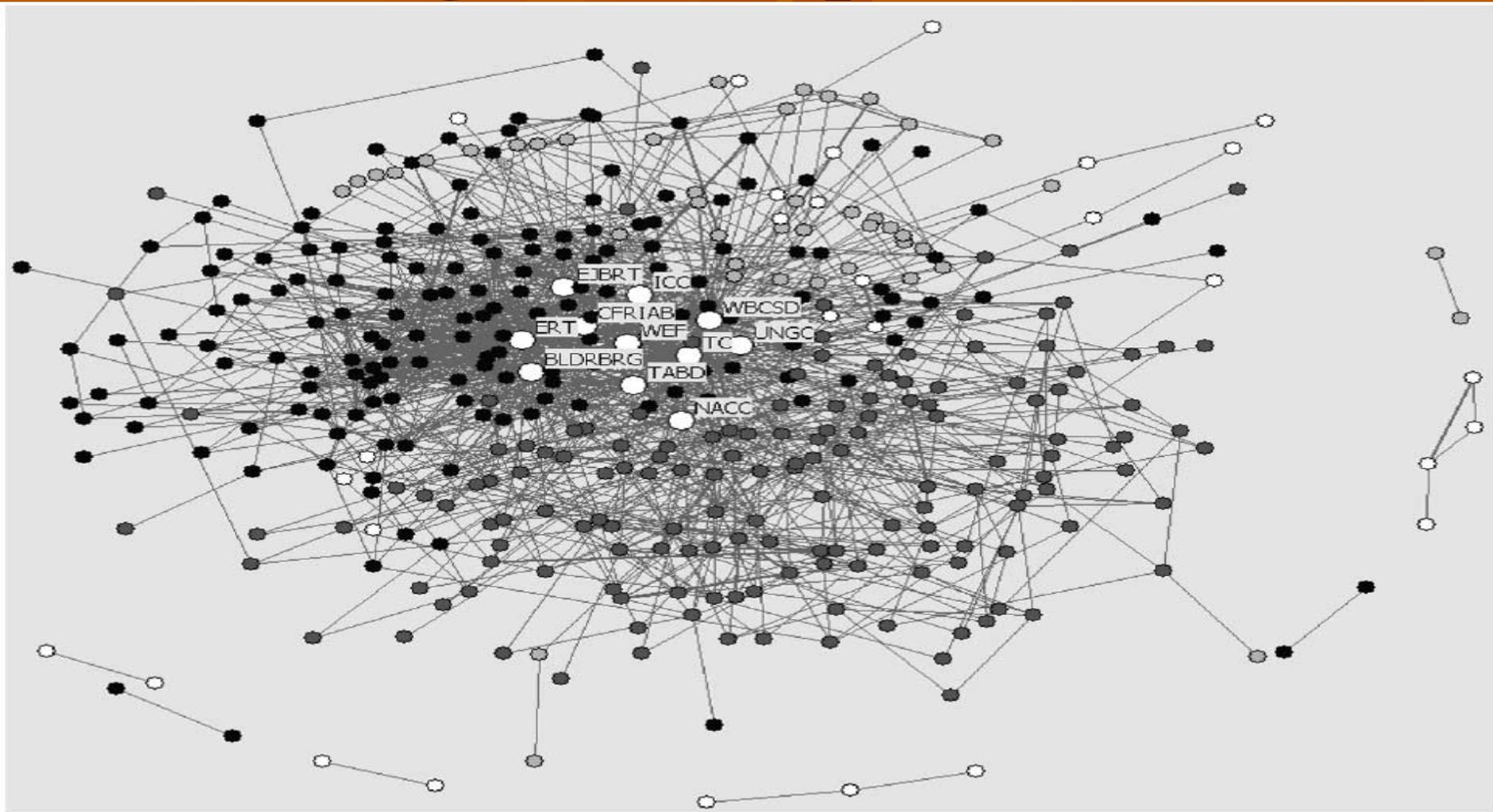


Figure 2 Twenty-Seven Key Players and Their Organizational Affiliations, 1996



# The global corporate elite



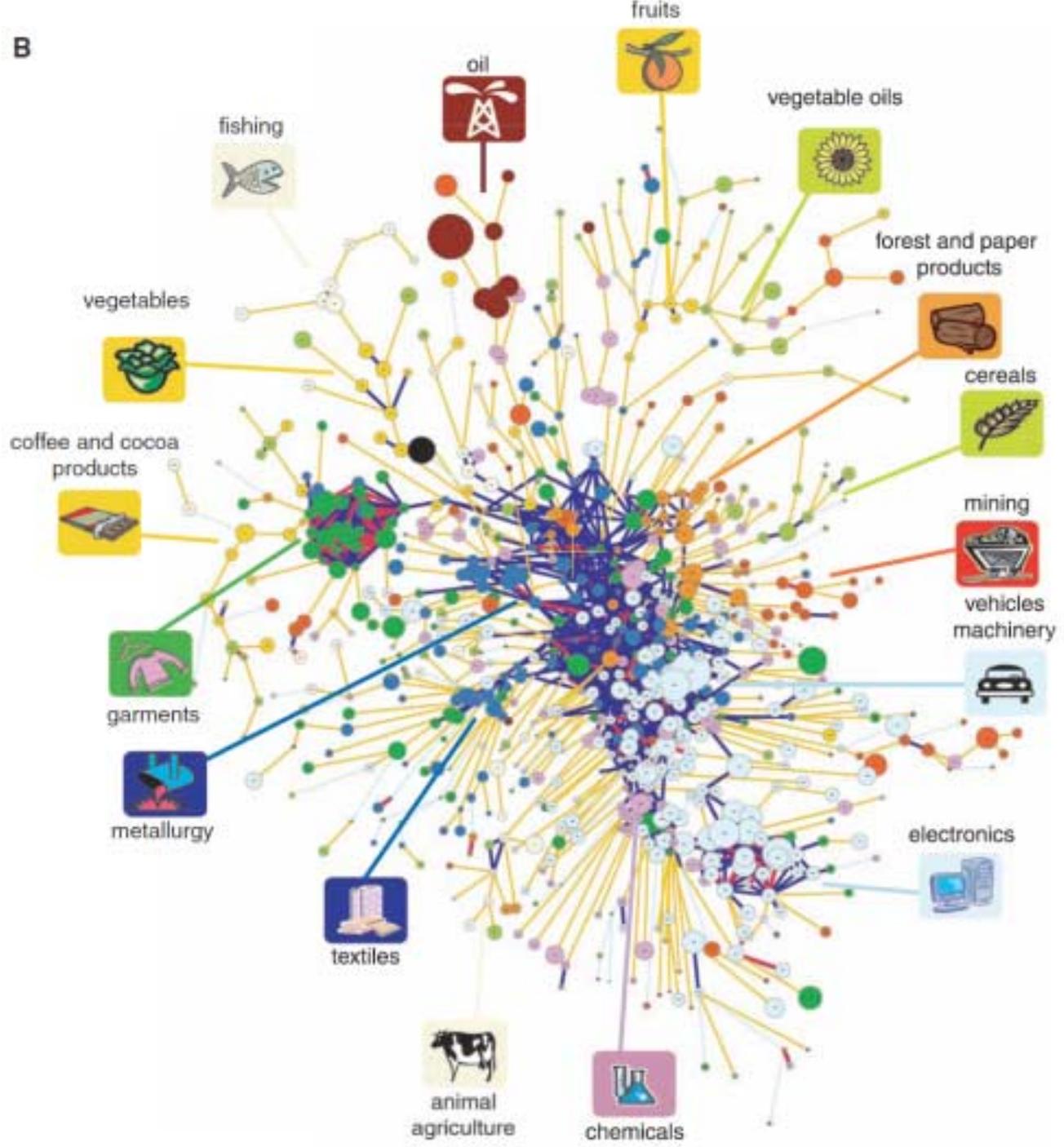
*Figure 4 Interorganizational Relations, 2006*

*Key:* White (large circles): policy boards; black: Europe; dark grey: US and Canada; light grey: Japan-Australia; white (small circles): rest of the world.

# Networks and the product space

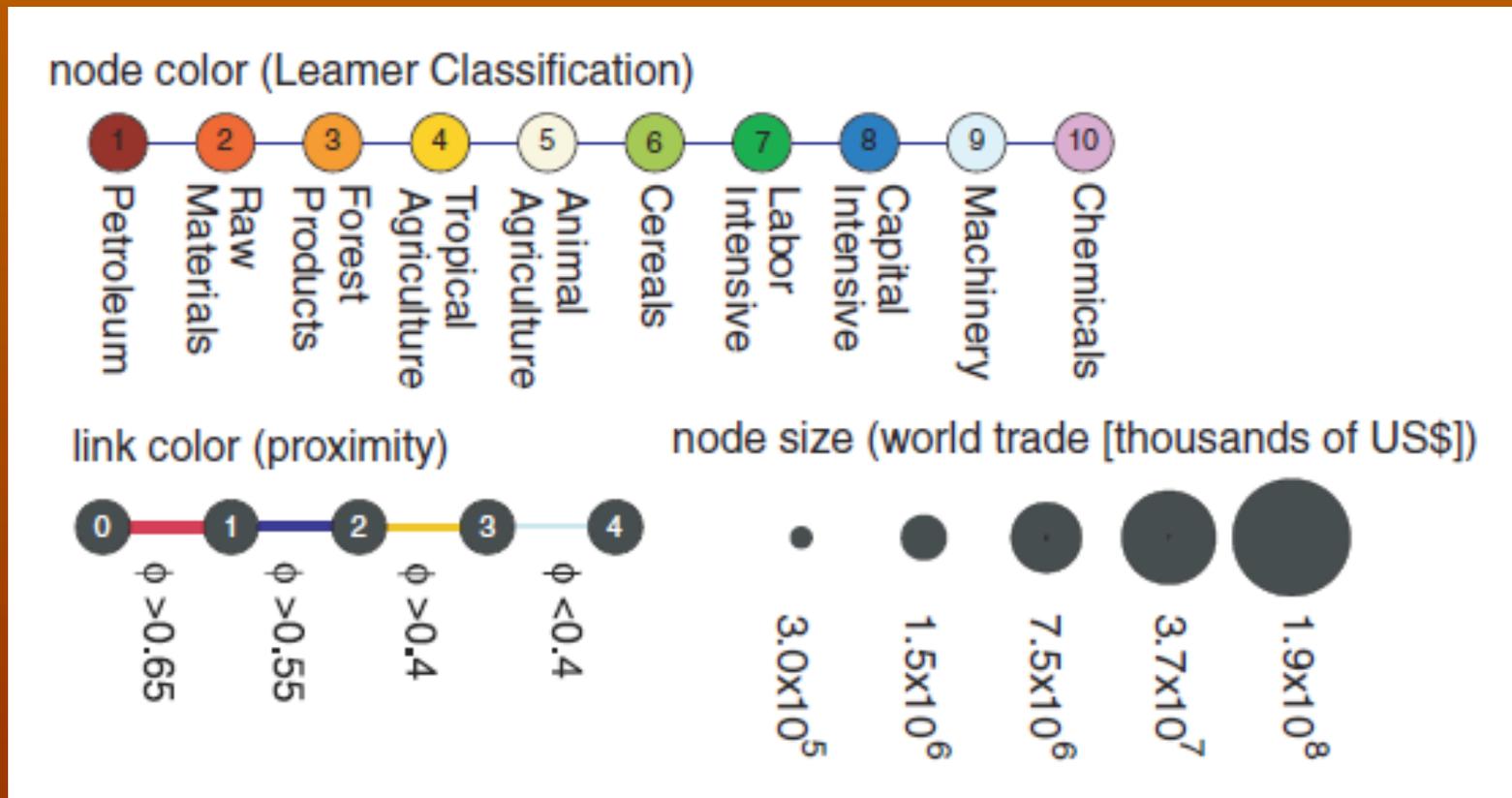
- Economies grow upgrading the products they produce and export
- Technology, capital and skills needed to make newer products are more easily adapted from some products than from others
- The network of relations between products, is called the “product space,”
- Sophisticated products are located in a densely connected core
- Less sophisticated ones occupy a less-connected periphery.
- Countries move through product space developing goods close to those they currently produce.
- To reach the core most countries need to move through large distances,
- Explains why poor countries have trouble developing competitive exports and converge to the income level of rich countries

B



# Networks and the product space

- Product codes, size and proximity



(C. A. Hidalgo, B. Klinger, A. L. Barabási and R. Hausmann, *Science* 317 (2007) 482-487)

# Models for the formation of strategic networks

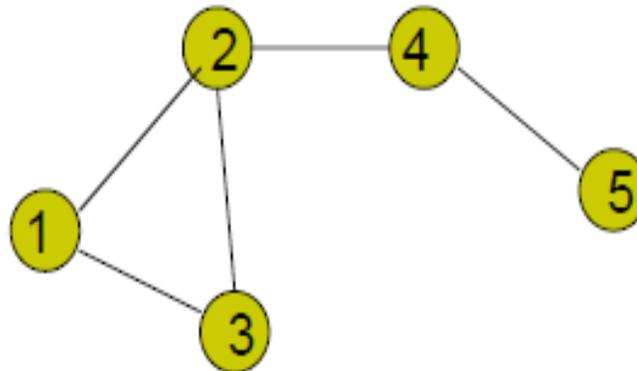
- In an economic context a network link is formed if and only if both agents (nodes) find that establishing that link is beneficial to them
- Therefore models require an utility function
- The notion of “**pairwise stability**” of a network  $g$  with links between agents  $i$  and  $j$  denoted  $ij$ 
  - $u_i(g) \geq u_i(g-ij)$  for each  $i$  and  $ij$  in  $g$
  - $u_i(g+ij) \geq u_i(g)$  implies  $u_j(g+ij) \geq u_j(g)$  for each  $ij$  not in  $g$
- Different from Nash equilibrium
- Efficient network when the total utility is maximum

# The connections model

Jackson and Wolinsky (1996):

- benefit from a friend is  $\delta$
- benefit from a friend of a friend is  $\delta^2, \dots$
- cost of a link is  $c$

$$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$$



$$u_2 = 3\delta + \delta^2 - 3c$$

$$u_5 = \delta + \delta^2 + 2\delta^3 - c$$

# The connections model

## Efficient Networks



- low cost:  $c < \delta - \delta^2$ 
  - complete network is efficient
- medium cost:  $\delta - \delta^2 < c < \delta + (n-2)\delta^2/2$ 
  - star network is efficient
    - minimal number of links to connect
    - connection at length 2 is more valuable than at 1 ( $\delta - c < \delta^2$ )
- high cost:  $\delta + (n-2)\delta^2/2 < c$ 
  - empty network is efficient

# The connections model

## Pairwise Stable Networks:

- low cost:  $c < \delta - \delta^2$ 
  - complete network is pairwise stable (and efficient)
- medium/low cost:  $\delta - \delta^2 < c < \delta$ 
  - star network is pairwise stable (and efficient)
  - others are also pairwise stable
- medium/high cost:  $\delta < c < \delta + (n-2)\delta^2/2$ 
  - star network is not pairwise stable (no loose ends)
  - nonempty pairwise stable networks are over-connected and may include too few agents
- high cost:  $\delta + (n-2)\delta^2/2 < c$ 
  - empty network is pairwise stable (and efficient)



# **Innovation and self-organization**



# Innovation and self-organization

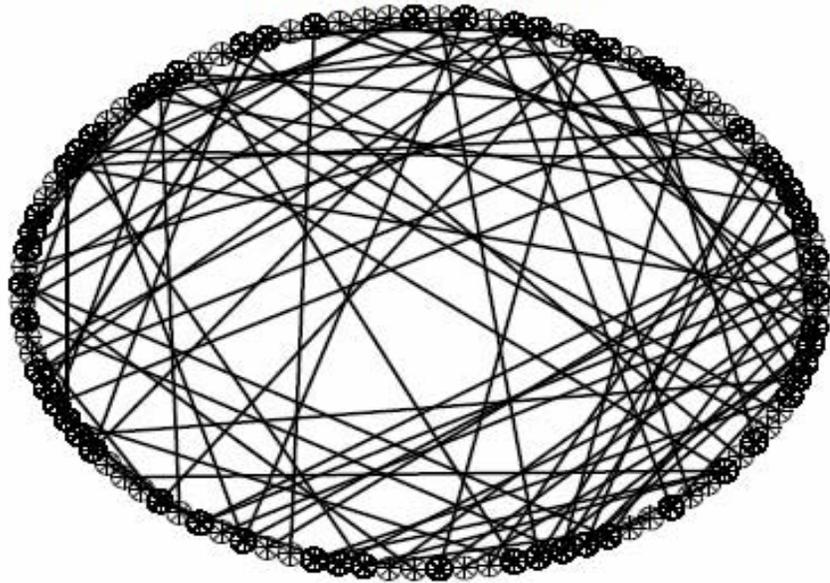
- A multi-agent model
- Each agent is characterized by two bit strings
- P-string: What the agent extracts from the environment (the other agents)
- N-string: What the other agents may extract from him.
- The model applies both to an economy or an ecological context
- Fitness of each agent

$$F_i(t + 1) = F_i(t) + \sum_{j(i)} \frac{q_{ij}^*}{k} - \frac{q_{l(i)i}^*}{k}.$$

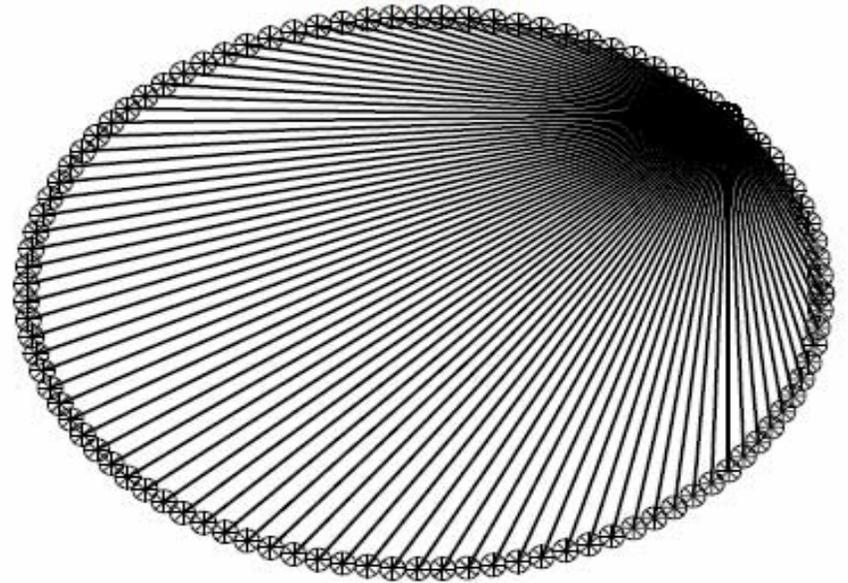
# Innovation and self-organization

- At each step of the evolution each agent matches his P string to the N strings of the other agents.
- Then, among those P-strings with the higher matching with a particular N-string, one is chosen at random that supplies (economy) or preys (ecology) the agent with the corresponding N-string. The  $q$ 's in the fitness are the overlaps.
- P-innovation means to change each time one bit to increase matching with the N-strings
- N-innovation means to change the bits to decrease the matching, therefore reducing what is given to the matching P-string.
- Supply (Economy) or Predation (Ecology) networks

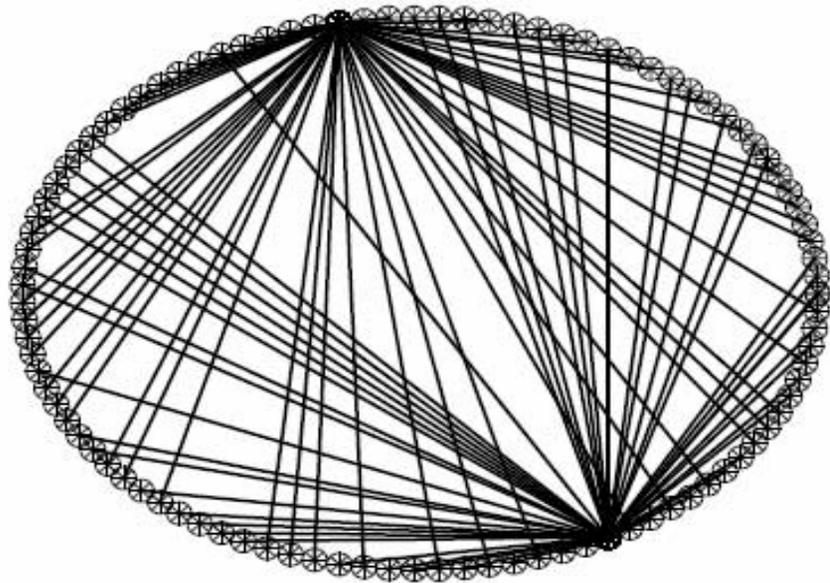
Without Innovation



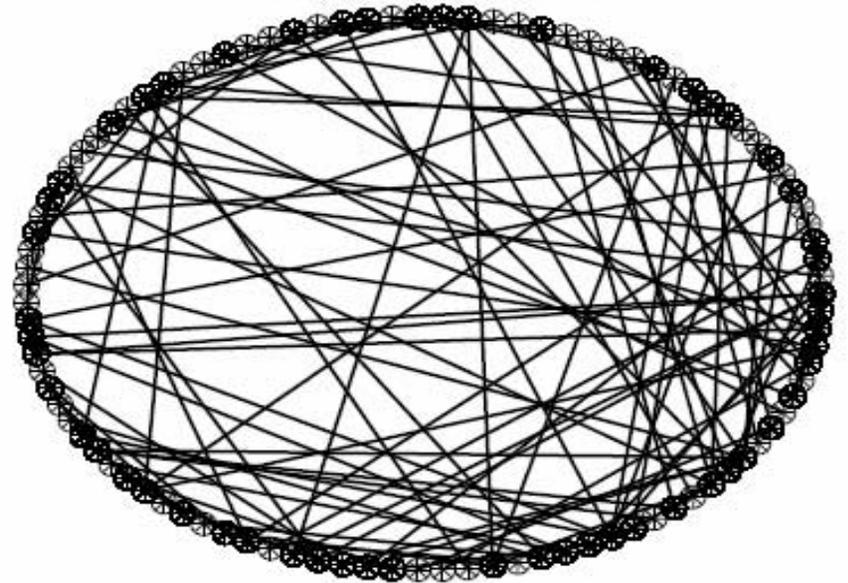
*P*-adaptation



*N*-adaptation



*P*- and *N*- adaptation



# Innovation and self-organization

- Conclusions:
- With P-innovation alone: a winner(s)-take-all situation
- With N-innovation alone: diversified suppliers, low cost
- With both P- and N-innovation: similar to the without innovation case

*(T. Araújo and RVM, Advances in Complex Systems 12 (2009) 233-253)*

# The stock market: An undirected weighted network

**Nodes: Companies**

**Links: established by a metric depending on the fluctuation correlations**

*RVM, T. Araújo and F. Louçã, Physica A 323 (2003) 625-648*

*T. Araújo and F. Louçã, Quantitative Finance 7 (2007) 63-74*

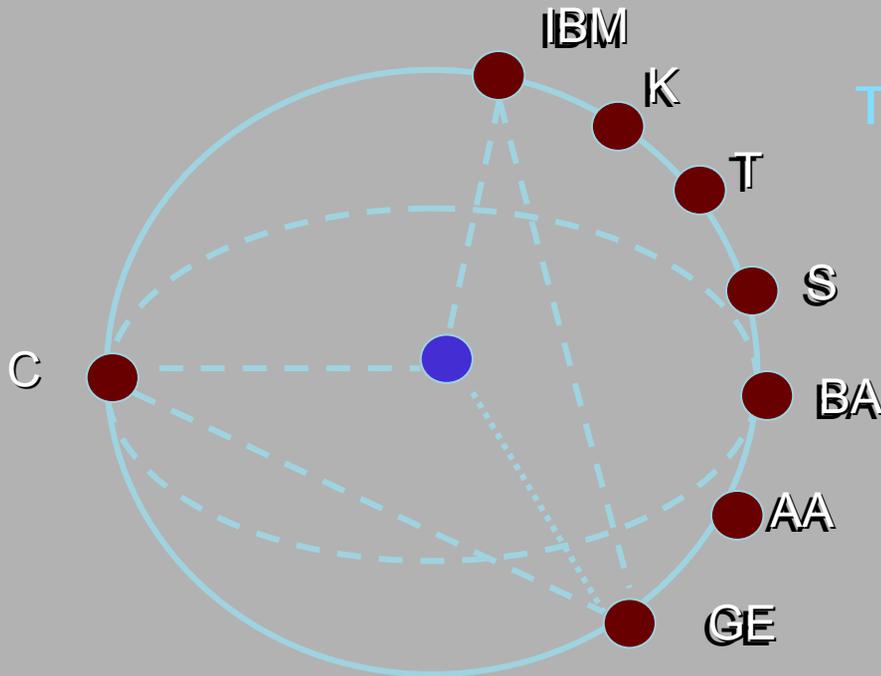
# Financial market geometry

r: return      $r(k) = \log(p_t(k)) - \log(p_{t-1}(k))$      p: price

## Metric

Distances defined from the returns correlation

$$d_{ij} = \sqrt{2(1 - c_{ij})}$$



## The market space

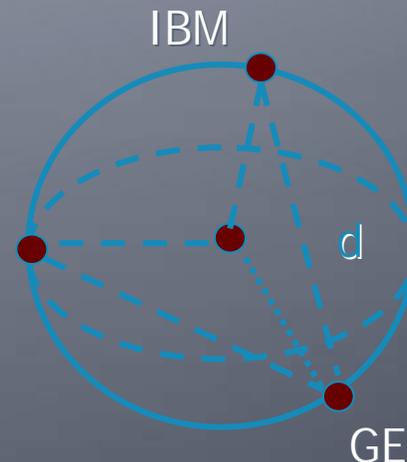
1. Compute each stock coordinates from the distances
2. Define the center of mass as the origin
3. Construct the inertia tensor
4. Identify the relevant  $f$  dimensions by comparison with a random permutation of the data

## The number of embedding dimensions

### S&P500 and Dow Jones, daily data

- 35 companies, 10 years
- 70 companies, 10 years
- 249 companies, 33 years
- 253 companies, 35 years
- 253 companies, 22 years
- 424 companies, 10 years

In all cases: No more than 6 dimensions !



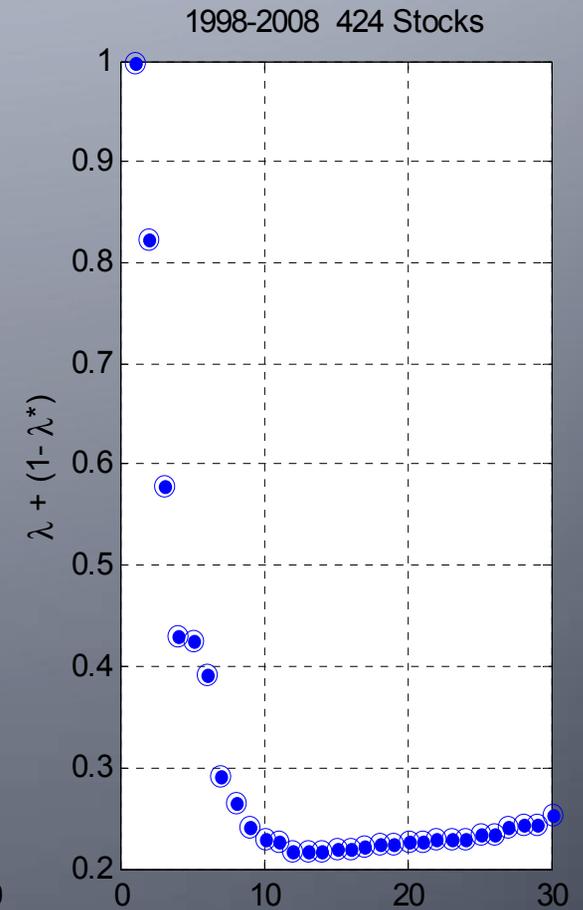
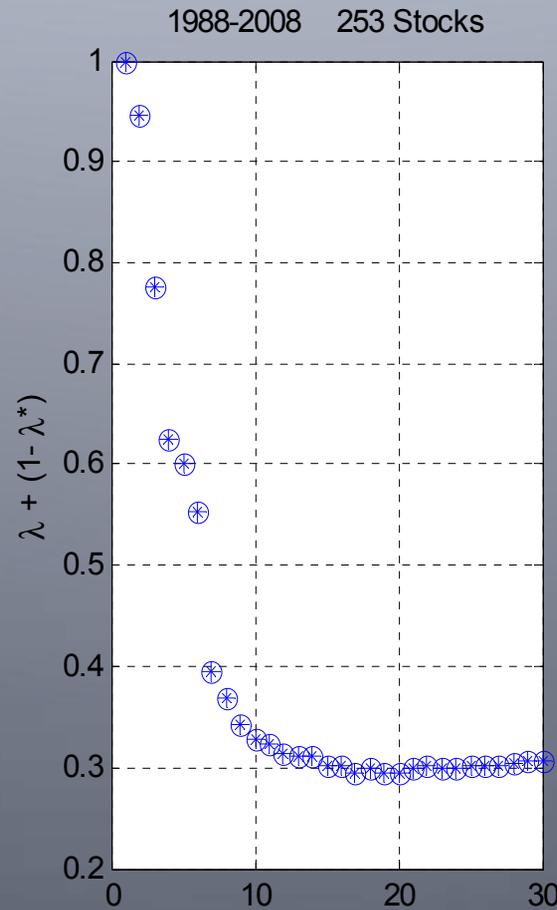
# Market spaces

## Eigenvalues compared with random permutation

$$\lambda + (1 - \lambda')$$

$\lambda$  : actual

$\lambda'$  : random



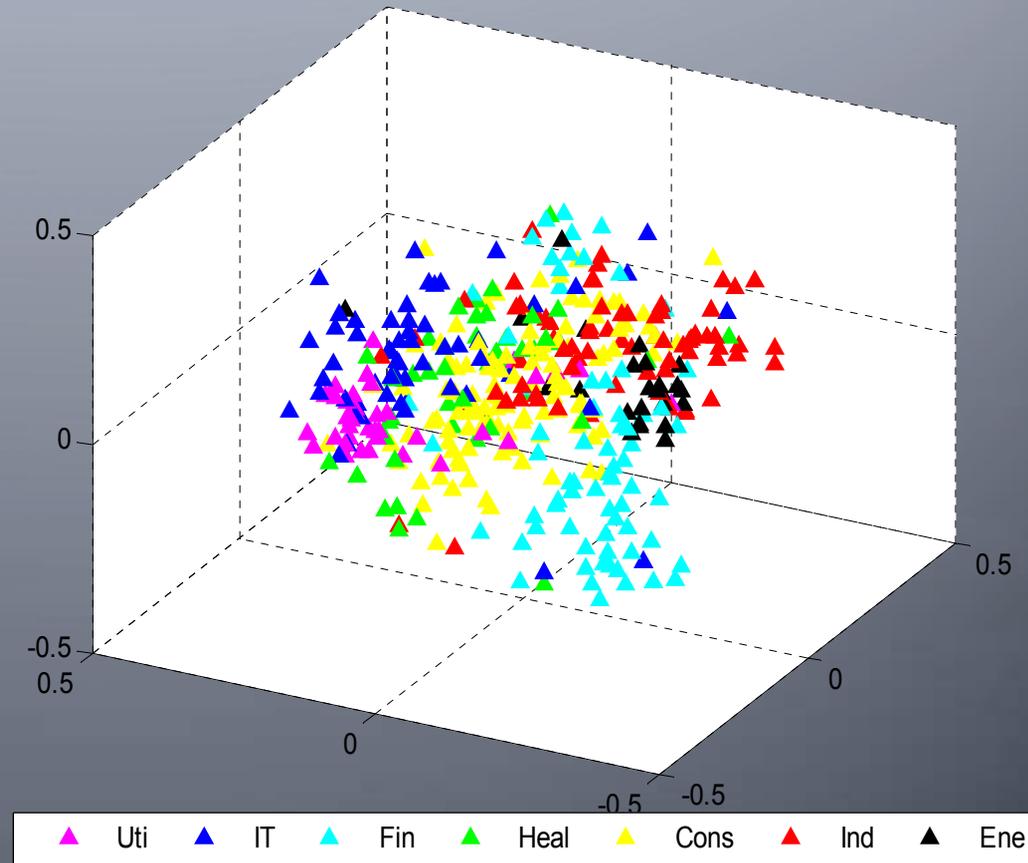
# Market spaces and crisis

## Shape modification at the crisis

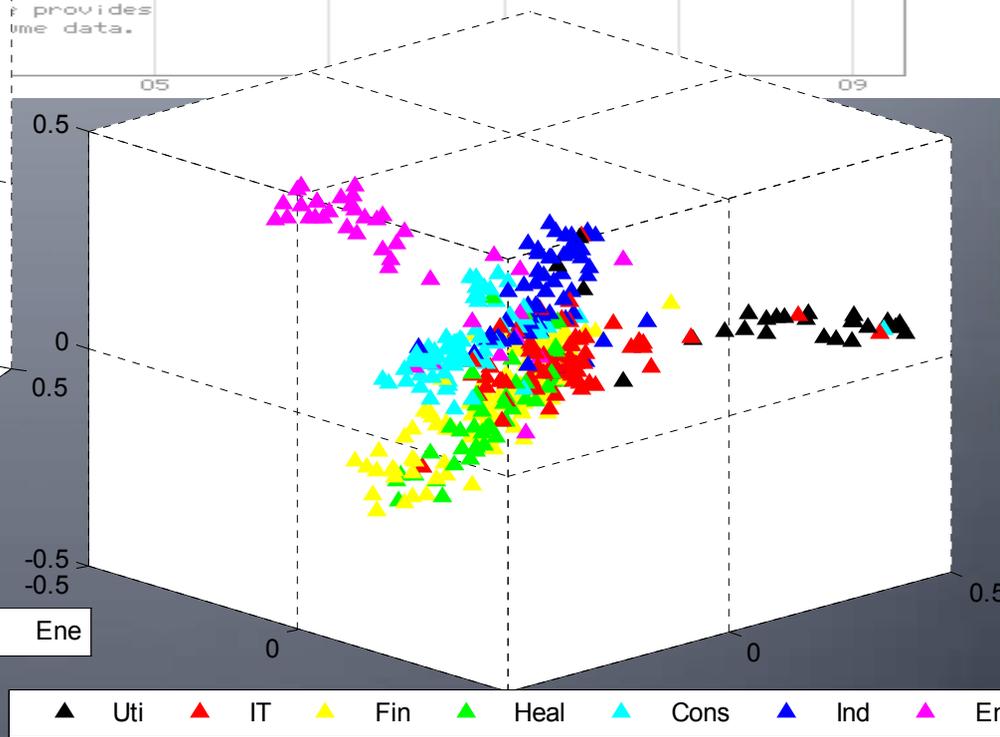
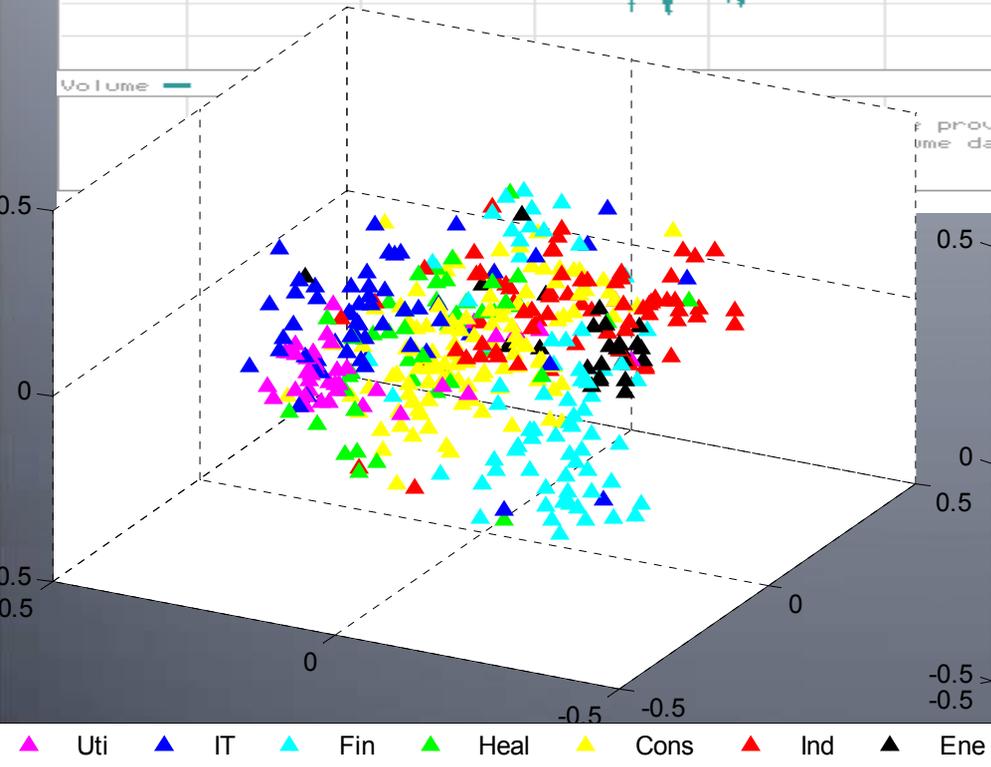
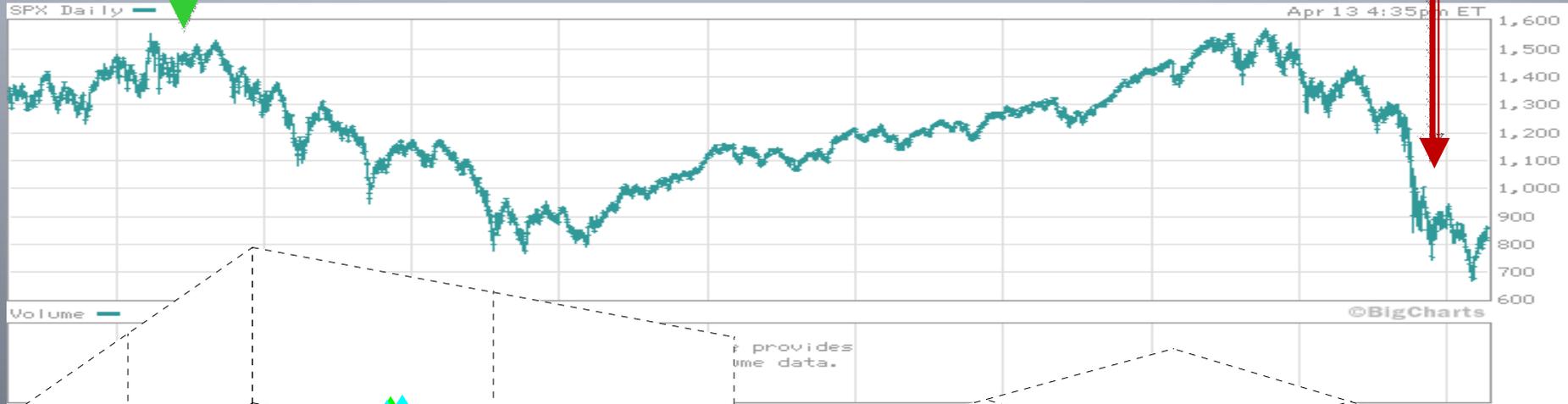
“Spherical” form

Typical of “surrogate data”  
and “business-as-usual”  
periods

Distortions and  
reduction of the  
distances during crisis



# Market shape: S&P500



# Structure index

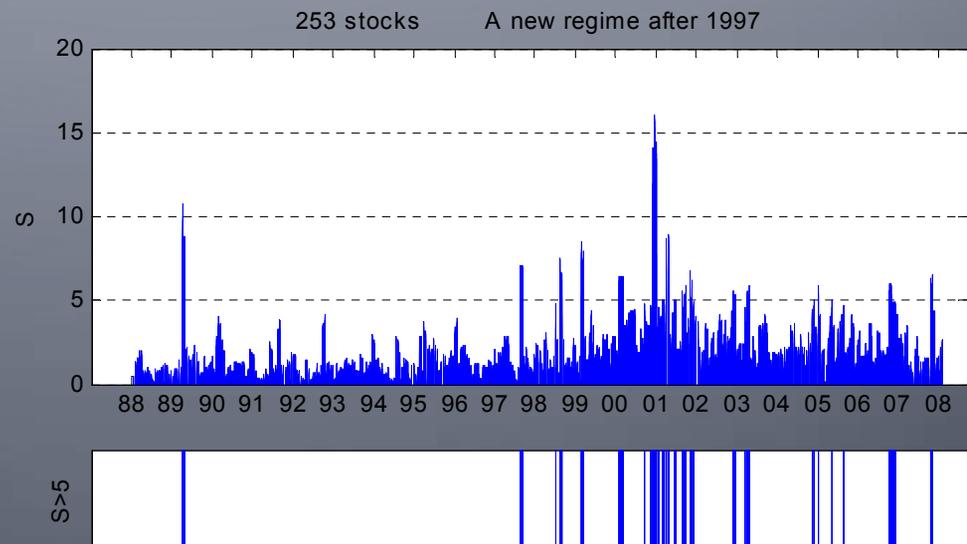
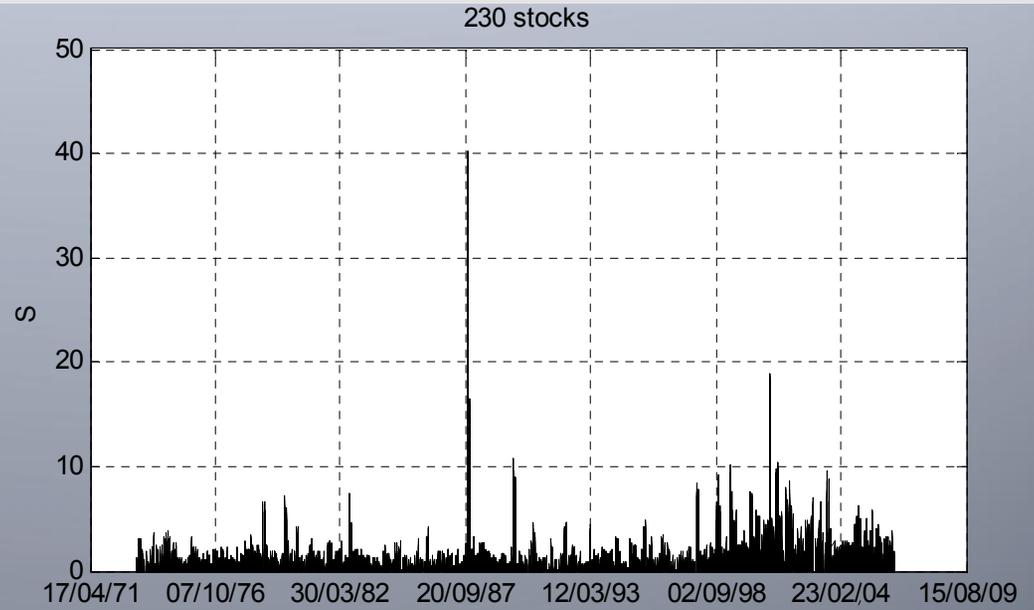
## Shape distortions

### Structure index

$\lambda$  : actual  
 $\lambda'$ : random

$$S_t = \sum_{i=1}^6 \left( \frac{\lambda_t'(i)}{\lambda_t(i)} - 1 \right)$$

After 1997 there are many periods with market distortions



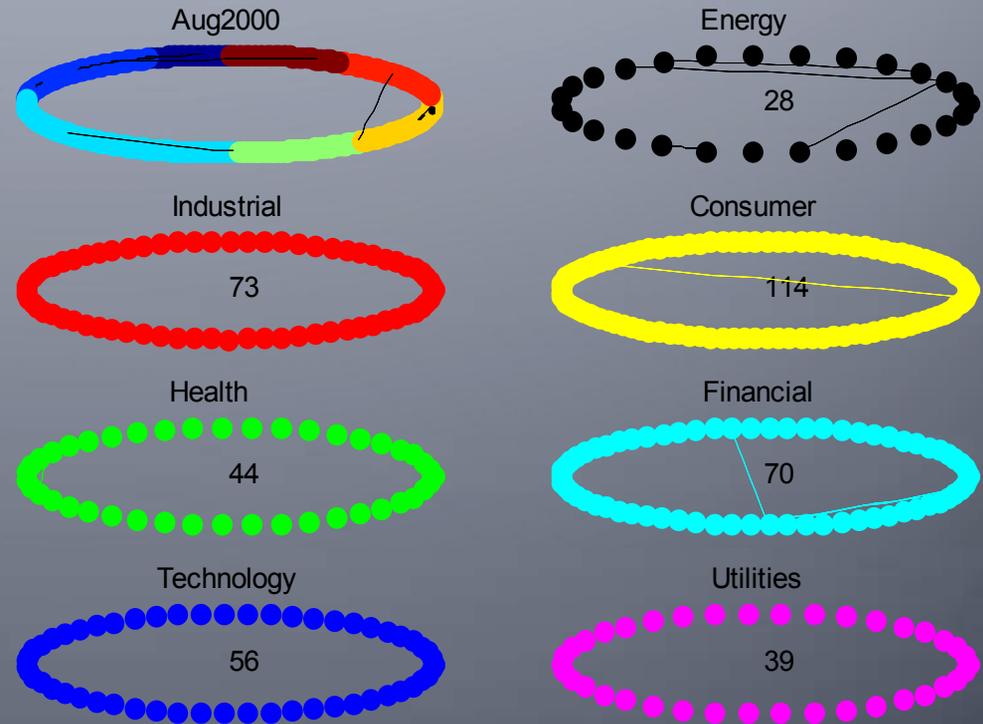
# Stock market networks

The market networks are weighted and fully connected

1. hierarchical clustering
2. minimal spanning tree
3.  $L_D^6$  smaller distance in  $R^6$  which insures network connectivity
4. Then

$$d^6_{i,j} \leq \frac{L_D^6}{2} \Rightarrow b_{i,j} = 1$$

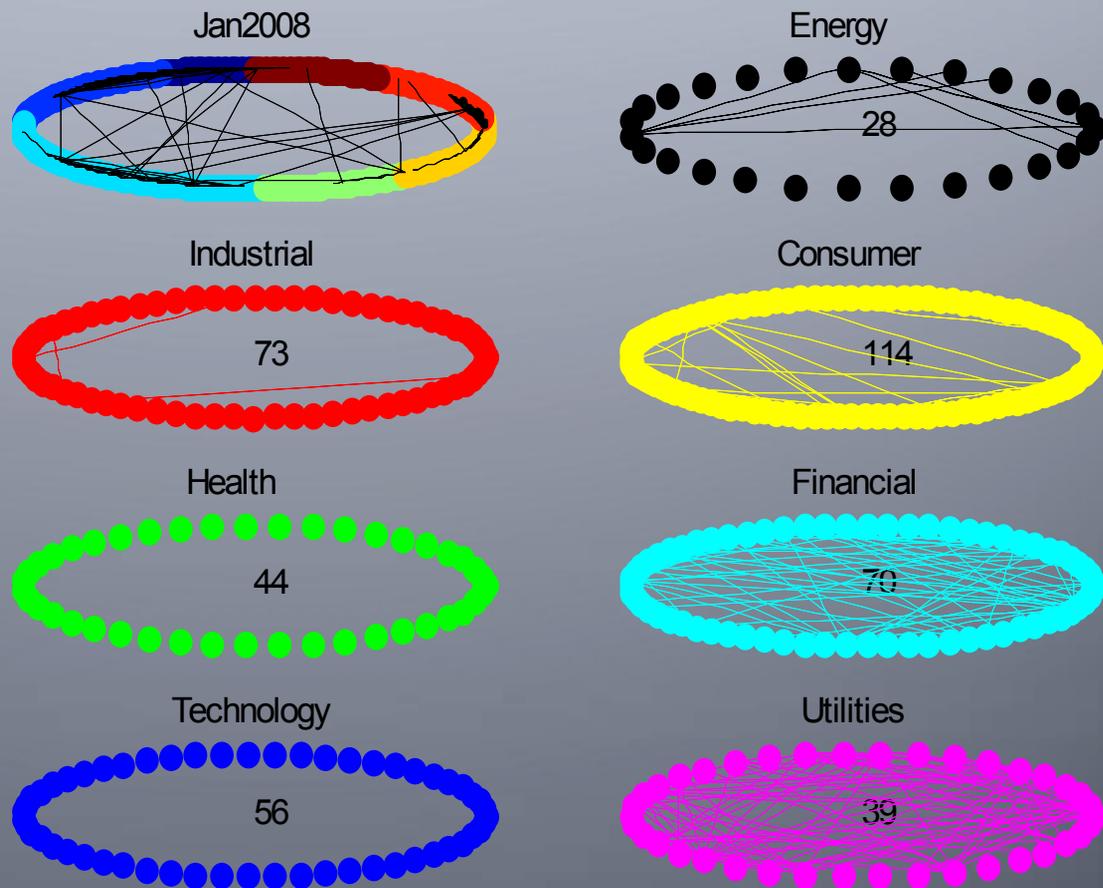
$$d^6_{i,j} > \frac{L_D^6}{2} \Rightarrow b_{i,j} = 0$$



Normal periods have few links

# Stock market networks

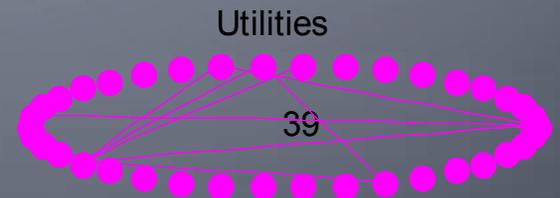
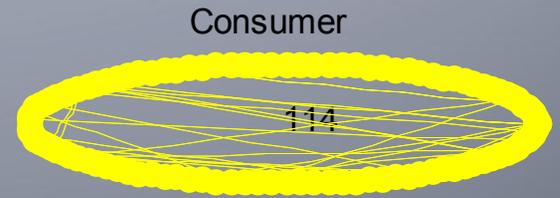
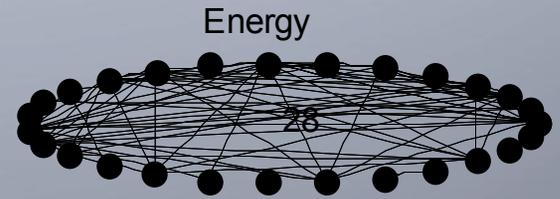
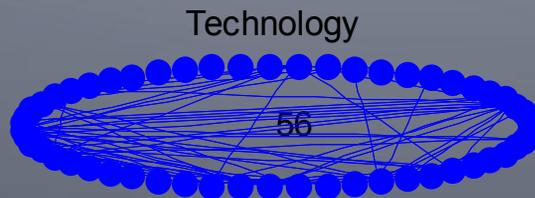
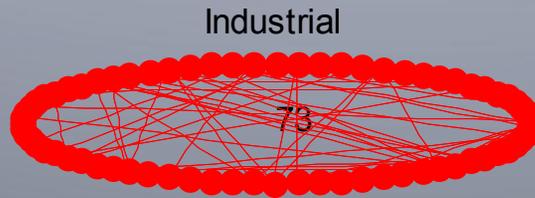
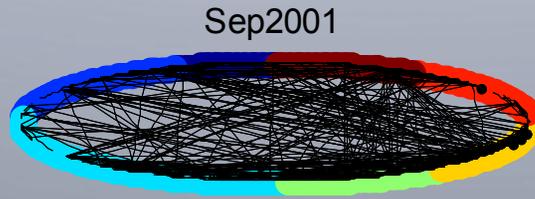
During the crisis the agents display higher correlations



Crisis periods: increase of the number of links, mostly inside the sectors

# Stock market networks

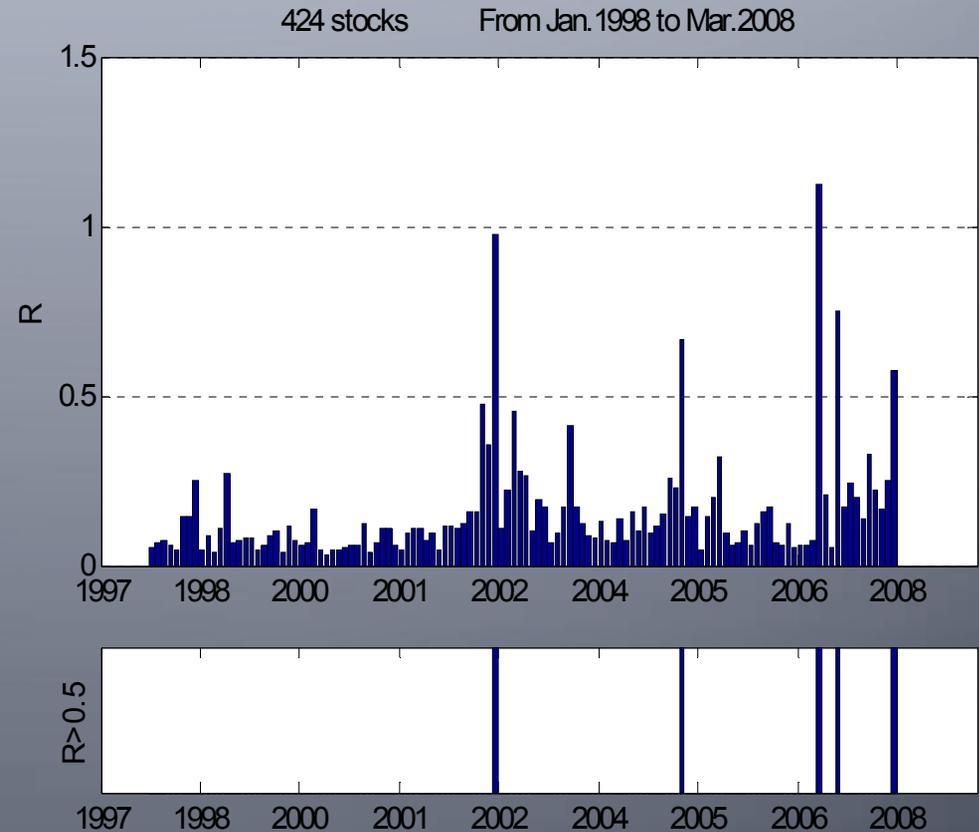
In some crisis:  
general  
increase of  
the number  
of links in all  
sectors



## Strong versus weak links

1. hierarchical clustering
2. minimal spanning tree
3.  $L_D^6$
4. Strong-weak ratio

$$R_t = \frac{\sum_{d_t^6(i,j) \leq L_D^6} d_t^6(i,j)}{\sum_{d_t^6(i,j) > L_D^6} d_t^6(i,j)}$$



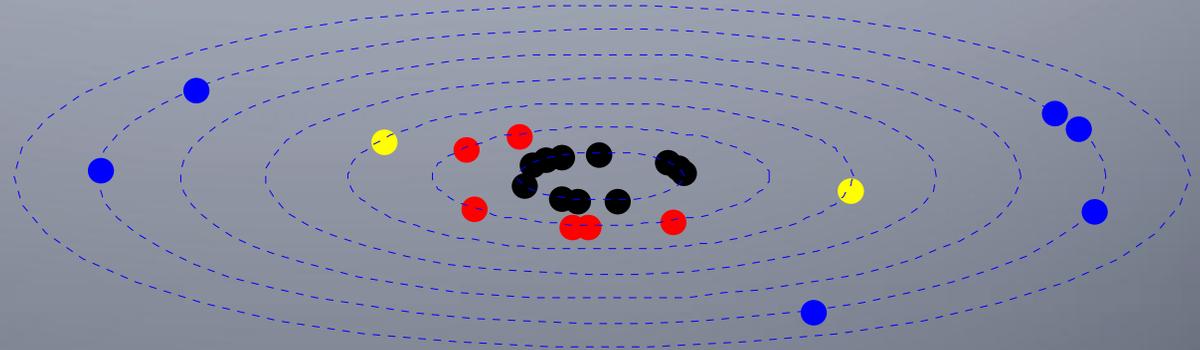
# States

## Synchronization (states)

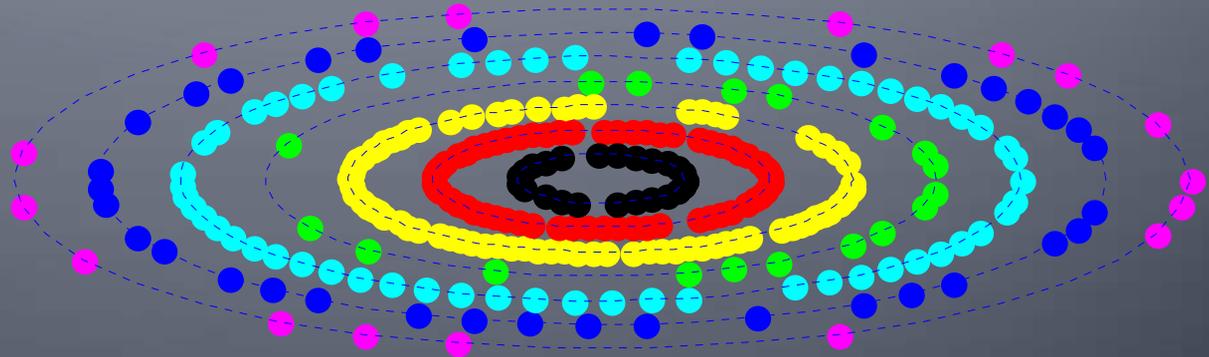
$$s_i = 1 \Leftrightarrow \exists d_t^6(i, j) \leq \frac{L_{D^6}}{2}$$

$s_i = 0$  otherwise

Mar1998



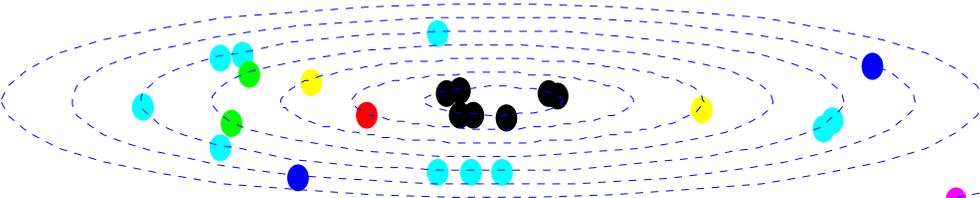
Sep2001



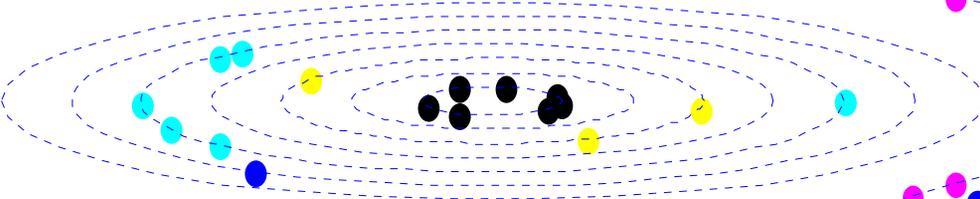
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# States

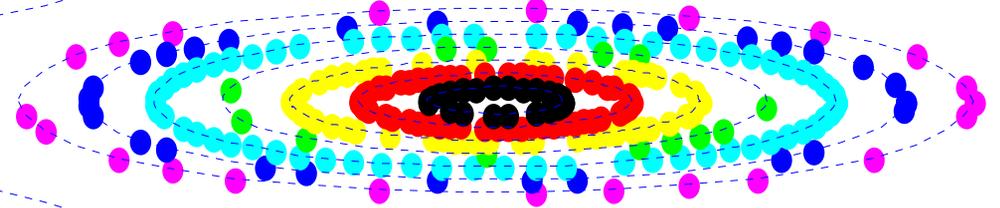
Aug2000



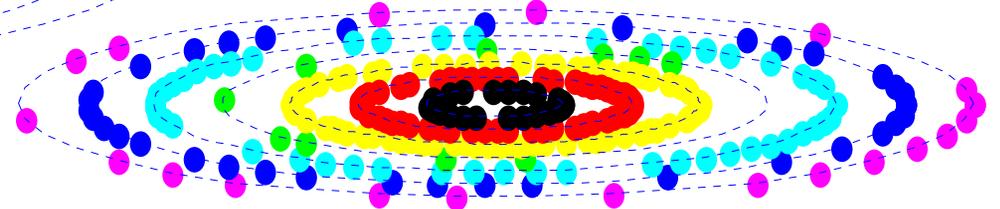
Sep2000



Dec2007



Jan2008



Feb2008

