T violation and the dark sector

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Abstract

It is argued, as a working hypothesis, that "normal" and dark matter interactions can only be T and CP violating. One way to implement this idea is to consider that time reversal in dark matter is implemented, not by an antiunitary operator, but by an unitary operator. It is shown how this occurs naturally in the context of complex spacetime with an extended symmetry group.

1 The interactions of dark matter

There is now extensive evidence for the existence in the Universe of matter not seen by our instruments. Strong evidence comes from the spiral galaxy rotation curves measured by visible and radio wavelengths, from the galaxy clusters matter content being larger than expected from luminosity, from the virial theorem, from velocity dispersion and gravitational lensing, as well as from galaxy clusters dynamics, the large scale structure and the cosmic microwave background radiation anisotropies, etc. Another current cosmological mystery is the acceleration of the universe expansion, which may be associated to an essentially constant density of dark energy. This may or may not be associated to the dark matter phenomena. Or it may simply mean that Einstein's equation with the cosmological term is the right classical gravitational equation. In any case matter in the dark sector, whatever it is, has so far manifested itself only by gravitational interactions?

A feature that might distinguish gravity from the other interactions at the quantum level, is the absence of global symmetries. It is of course possible to construct quantum field theory Lagrangians for matter and gravitational fields which, at the perturbative level, preserve all symmetries. However it has for a long time been conjectured [1] [2] that, in an actual quantum theory of gravity, no global symmetries will be preserved. This conjecture which, in the past, was mostly based on singularity and black hole physics was somewhat strengthened

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in the AdS/CFT holographic context [3]. Admittedly AdS/CFT is not the final quantum gravity theory, nevertheless it is, so far, the best understood consistent framework.

Among the symmetries that would not be preserved in quantum gravity is time-reversal ([3], Sec.7 and Th. 4.2). This suggests as a possible interpretation (or a working hypothesis) for the elusive nature of the interactions of dark with ordinary matter, that they must be T-violating interactions. If CPT is conserved, at least locally, this might also imply a CP-violating nature for the interactions of dark and ordinary matter.

That only T and CP violating interactions are associated to dark matter is a general hypothesis that might be realized in different ways. Here a particular scenario will be explored, in a complex spacetime context, where normal and dark matter belong to different superselection sectors, having different symmetry properties.

2 T-violation in a complex spacetime scenario

Several authors have studied extensions of gravitational theory by complexifying the functions that appear in the classical equations. Already Einstein in 1945 [4] generalized his theory of gravitation by considering a metric with complex components and Hermitian symmetry, others [5] have complexified the tangent bundle over the real four-dimensional spacetime manifold, constructed solutions of the complex Einstein equation [6], or provided a complex manifold description of massless particles [7] [8], etc.

A more radical approach consisted in assuming that the spacetime manifold itself should be parametrized but complex coordinates. Several such explorations have been carried out, see for example [9] [10] and for a more complete set of references and perspectives see Ref.[11]. Of course, a question that is left open is, if the spacetime dimensions are indeed indexed by complex numbers, why we do not feel it in our everyday experience.

Recently this question was revisited [12] for spacetimes where coordinates take values in higher normed division algebras, complex, quaternion or octonion. In this extended spacetime, any four independent directions may be chosen as a basis for a real vector space, that is, for a *real slice* of the complex spacetime. Using as consistency condition that the symmetry groups of the extended spacetime must reduce to the real Lorentz and Poincaré groups in each real slice, one obtains, for the homogeneous group of the extended spacetime, the condition

$$\Lambda^{\dagger}G\Lambda = G \tag{1}$$

that is, a $U(1,3,\mathbb{K})$ group, $\mathbb{K} = \mathbb{C}, \mathbb{Q}, \mathbb{O}$, with G the metric (1,-1,-1,-1). For $\mathbb{K} = \mathbb{C}$ this has been called the complex Lorentz group with real metric [13]. It is a 16-parameter group different from the 12-parameter group

$$\Lambda^T G \Lambda = G \tag{2}$$

used in the analytic continuations of the S-matrix. Nevertheless it is the condition (1) that insures an identical group in each one of the real slices.

The nature of the matter states, that is implied by this group structure, was obtained by studying the representations of the semidirect group

$$T_4 \circledast U\left(1, 3, \mathbb{C}\right) \tag{3}$$

 T_4 being the complex spacetime translations. The conclusions were:

- Half-integer spin states are not elementary states for the full complex group (3). That is, half-integer elementary states, that are irreducible representations of the real groups, cannot communicate between different real slices, in the sense that they cannot be rotated from one real slice to another.

- Integer-spin states are elementary states of the full group. However these states have a particular nature when the full group, including reflections, is taken into account. In the complex Lorentz group (1) parity and time reversal are continuously connected to the identity. Therefore, in faithful continuous norm-preserving representations, both parity and time reversal must be implemented by unitary operators. On the other hand one knows that, in each real slice, positivity of energy requires an antiunitary time reversal operator.

In conclusion: In the complex spacetime scenario there are three types of states: half-integer spin states that are confined to the real slices, integer-spin positive energy states that are also only associated to the real slices and finally integer-spin states, with unitary time reversal operator, that can communicate between real slices. Because of its association to time reversal violation, this last type will be denoted here as *chronoparticles*.

The conclusions are qualitatively similar for the normed division algebras \mathbb{Q} and \mathbb{O} [12]. Also notice that although the extended Lorentz group does not contain half-integer elementary states, it might still operate on these states. However, rather than rotating them between real slices it generates a multiplicity of states (see [12], Appendix D).

It is worthwhile to recall the energy positivity argument that leads to the antiunitarity of the time reversal operator. When time-reversal is a conserved symmetry, invariance of the Schrödinger equation

$$i\partial_t \psi = H\psi$$

implies that either the T operator is antiunitary or it anticommutes with the Hamiltonian. However, if it is unitary and anticommutes with the Hamiltonian, then

$$(T\psi, HT\psi) = -(\psi, H\psi)$$

and there are negative energy states.

Chronoparticles, as defined above, because they connect different real slices may establish interactions between matter in different real slices. Because of the argument above, they might behave as negative energy particles from the point of view of physics in the real slices. However, the above reasoning does not really apply because T is not conserved in their interactions. Because of the unitarity of the T operator, chronoparticles and ordinary matter in the real slices are in different superselection sectors and in their interactions T is not a symmetry. The argument [12] goes as follows:

Let $\psi_o \in V_o$ be a state in the (real slice) ordinary matter space and $\psi_c \in V_c$, a chronoparticle (dark matter?) state. There is a superselection rule operating between these two types of spaces. Consider a linear superposition of two of these states

$$\Phi = \alpha \psi_o + \beta \psi_c$$

with α, β real numbers. Now Φ and $e^{i\theta}\Phi = \alpha e^{i\theta}\psi_o + \beta e^{i\theta}\psi_c$ belong to the same ray and therefore should represent the same state. Applying the time reversal operator to both Φ and $e^{i\theta}\Phi$

$$T\Phi = \alpha T\psi_o + \beta T\psi_c$$

$$Te^{i\theta}\Phi = \alpha e^{-i\theta}T\psi_o + \beta e^{i\theta}T\psi_c$$
(4)

 $T\Phi$ and $Te^{i\theta}\Phi$ belong to different rays, hence T does not establish a ray correspondence in $V_o \oplus V_c$ unless $\alpha = 0$ or $\beta = 0$, that is, V_o and V_c belong to different superselection sectors.

The fact that the states are in different superselection sectors does not mean that they cannot interact. Whereas the superselection rule result concerns the structure of the direct sum $V_o \oplus V_c$, the nature of the interactions depends on the way the group transformations operate in the tensor product $V_o \otimes V_c$. Consider now the T operation acting on $V_o \otimes V_c$ and compute its action on a matrix element

$$\left(T\left(\psi_o^{(1)}\otimes\psi_c^{(1)}\right), T\left(\psi_o^{(2)}\otimes\psi_c^{(2)}\right)\right) = \left(U_o\psi_o^{(2)}\otimes U_c\psi_c^{(1)}, U_o\psi_o^{(1)}\otimes U_c\psi_c^{(2)}\right)$$
(5)

where U_o and U_c are unitary operators, acting on the degrees of freedom of the states. From (5) one concludes that there is no choice of phases that can make T a global unitary or anti-unitary operator in the tensor product space. Therefore, by Wigner's theorem, T cannot be a symmetry in $V_o \otimes V_c$.

How a particle with a unitary implementation of the T operation may induce symmetry violations in its interactions with ordinary matter is simple to illustrate. Let the Fourier decompositions of the quantum fields Φ_o and Φ_c , associated to ordinary matter and to an integer-spin chronoparticle be

$$\Phi_{o}\left(x^{0}, \overrightarrow{x}\right) = \sum_{\mu} \int d^{3}k \frac{\epsilon\left(k, \mu\right)}{2\omega\left(k\right)} \left\{a_{o}\left(k, \mu\right) e^{-ik.x} + b_{o}^{\dagger}\left(k, \mu\right) e^{ik.x}\right\}$$

$$\Psi_{c}\left(x^{0}, \overrightarrow{x}\right) = \sum_{\mu} \int d^{3}k \frac{\epsilon\left(k, \mu\right)}{2\omega\left(k\right)} \left\{a_{c}\left(k, \mu\right) e^{-ik.x} + b_{c}^{\dagger}\left(k, \mu\right) e^{ik.x}\right\}$$
(6)

 μ being an helicity label and $\epsilon\left(k,\mu\right)$ the polarization vectors. With

$$P\Phi\left(x^{0}, \overrightarrow{x}\right)P^{-1} = \eta_{P}\Phi\left(x^{0}, -\overrightarrow{x}\right)$$
$$T\Phi\left(x^{0}, \overrightarrow{x}\right)T^{-1} = \eta_{T}\Phi\left(-x^{0}, \overrightarrow{x}\right)$$
$$C\Phi\left(x^{0}, \overrightarrow{x}\right)C^{-1} = \eta_{C}\Phi^{\dagger}\left(x^{0}, \overrightarrow{x}\right)$$
(7)

for both ordinary matter and chronoparticle fields, one obtains

$$Pa_{o}\left(k^{0}, \overrightarrow{k}\right)P^{-1} = \eta_{P}a_{o}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Pb_{o}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)P^{-1} = \eta_{P}b_{o}^{\dagger}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Ta_{o}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{T}a_{o}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Tb_{o}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{T}b_{o}^{\dagger}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Ca_{o}\left(k^{0}, \overrightarrow{k}\right)C^{-1} = \eta_{C}b_{o}\left(k^{0}, \overrightarrow{k}\right)$$

$$Cb_{o}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{C}a_{o}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)$$
(8)

for ordinary matter, with ${\cal P}$ and ${\cal C}$ unitary and ${\cal T}$ antiunitary and

$$Pa_{c}\left(k^{0}, \overrightarrow{k}\right)P^{-1} = \eta_{P}a_{c}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Pb_{c}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)P^{-1} = \eta_{P}b_{c}^{\dagger}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Ta_{c}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{T}b_{c}^{\dagger}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Tb_{c}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{T}a_{c}\left(k^{0}, -\overrightarrow{k}\right)$$

$$Ca_{c}\left(k^{0}, \overrightarrow{k}\right)C^{-1} = \eta_{C}a_{c}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)$$

$$Cb_{c}^{\dagger}\left(k^{0}, \overrightarrow{k}\right)T^{-1} = \eta_{C}b_{c}\left(k^{0}, \overrightarrow{k}\right)$$
(9)

for chronoparticle operators, with P and T unitary and C antiunitary (for CPT invariance). For simplicity the helicity labels were dropped.

Now it is clear in which way chronoparticles violate T-invariance. Let the interaction term of ordinary matter with chronoparticles be

$$\mathcal{L}_I \sim \Phi_o^\dagger \Phi_o \Phi_c$$

Then, the simple vertex

$$\left\langle \phi_{o\left(\overrightarrow{k}+\overrightarrow{v}\right)} \left| a_{o}^{\dagger}\left(\overrightarrow{k}+\overrightarrow{v}\right) a_{c}\left(\overrightarrow{v}\right) a_{o}\left(\overrightarrow{v}\right) \right| \phi_{o\left(\overrightarrow{k}\right)} \psi_{c\left(\overrightarrow{v}\right)} \right\rangle$$

where an ordinary matter state $\phi_{o\left(\overrightarrow{k}\right)}$ absorbs a chronoparticle $\psi_{c\left(\overrightarrow{v}\right)}$ leading to $\phi_{o\left(\overrightarrow{k}+\overrightarrow{v}\right)}$, becomes, under the *T*-operation

$$\left\langle \phi_{o\left(-\overrightarrow{k}\right)}\psi_{c\left(-\overrightarrow{v}\right)}^{\dagger}\left|a_{o}^{\dagger}\left(-\overrightarrow{k}\right)b_{c}^{\dagger}\left(-\overrightarrow{v}\right)a_{o}\left(-\overrightarrow{k}-\overrightarrow{v}\right)\right|\phi_{o\left(-\overrightarrow{k}-\overrightarrow{v}\right)}\right\rangle$$

rather than

$$\left\langle \phi_{o\left(-\overrightarrow{k}\right)}\psi_{c\left(-\overrightarrow{v}\right)}\left|a_{o}^{\dagger}\left(-\overrightarrow{k}\right)a_{c}^{\dagger}\left(-\overrightarrow{v}\right)a_{o}\left(-\overrightarrow{k}-\overrightarrow{v}\right)\right|\phi_{o\left(-\overrightarrow{k}-\overrightarrow{v}\right)}\right\rangle$$

That is, the T-operation leads to a process different from the usual timereversed one and therefore does not establish an identification of the couplings for the direct and the reversed processes. Notice that in this example, because C is chosen to be antiunitary for chronoparticles, one has CT and P invariance but CP and T violation. Chronoparticles may induce T-violating interactions, either by establishing interactions between matter particles in different real slices or in each real slice by virtual chronoparticle interactions.

References

- R. Penrose; Singularities and time-asymmetry, in General Relativity: An Einstein Centenary Survey, S. W. Hawking and S. W. Israel (Eds.), pp. 581-638, Cambridge U. P., Cambridge 1979.
- [2] R. M. Wald; Quantum gravity and time reversibility, Phys. Rev. D 21 (1980) 2742-2755.
- [3] D. Harlow and H. Ooguri; Symmetries in Quantum Field Theory and Quantum Gravity, Comm. Math. Phys. 383 (2021) 1669–1804.
- [4] A. Einstein; A generalization of the relativistic theory of gravitation, Annals of Math. 46 (1945) 578-584.
- [5] G. Kunstatter and R. Yates; The geometrical structure of a complexified theory of gravitation, J. Phys. A: Math Gen. 14 (1981) 847-854.
- [6] J. Plebanski; Some solutions of complex Einstein equations, J. Math. Phys. 16 (1975) 2395-2402.
- [7] R. Penrose; The twistor approach to space-time structures, in 100 Years of Relativity, A. Ashtekar (Ed.) pp. 465-505, World Scientific, Singapore 2005.
- [8] G. Esposito; From spinor geometry to complex general relativity, Int. J. of Geometric Methods in Modern Physics 2 (2005) 675-731.
- [9] E. H. Brown; On the complex structure of the universe, J. Math. Phys. 7 (1966) 417-425.
- [10] E. T. Newman; Maxwell's equations and complex Minkowski space; J. Math. Phys. 14 (1973) 102-103.
- [11] G. Esposito; Complex geometry of Nature and general relativity, Kluwer Acad. Press, Dordrecht 2002.
- [12] R. Vilela Mendes; Space times over normed division algebras, revisited, Int. J. Modern Physics A 35 (2020) 2050055.
- [13] A. O. Barut; Complex Lorentz group with a real metric: Group structure, J. Math. Phys. 5 (1964) 1652-1656.