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Technical Briefs

Boundary-Layer Control by Electric Fields

R. Vilela Mendes[1] and J. A. Dente[2]

1 Boundary Layers and Boundary Layer Control

In turbulent boundary layers, near-wall streamwise vortices are responsible for the high skin friction drag. Therefore, most recent attempts (Sirovich and Karlsson, 1997; Lee, et al., 1997) to reduce the skin friction drag concentrate on the control of the interactions between the vortices and the wall. However, because of the very large ratio between laminar and turbulent skin friction drag, it also makes sense to devote some effort to delay the transition. The control techniques that have been proposed include suction of slow-moving fluid through slots or a porous surface, use of compliant walls and wall cooling (or wall heating for liquids). Another class of techniques for boundary-layer control consists in acting on the flow by means of electromagnetic forces. Here different techniques should be envisaged according to whether the fluid is weakly conducting (an electrolyte like seawater or an ionized gas) or a good conductor (like a liquid metal). Proposals for boundary-layer control by electromagnetic forces trace its origin to the papers of Gailitis and Lielausis (1961), Tsinober and Shtern (1967), and Moffat (1967). Interest in these techniques has revived in recent years and some more accurate calculations and experimental verifications have been carried out, mostly in the context of electrolyte fluids (Tsinober, 1990; Henoch and Stace, 1995).

In this note we are concerned with the flow of air along an airfoil when a layer of ionized gas is created on the boundary-layer region. Local ionization of the air along the airfoil not being practical from the technological point of view, we will assume that a stream of ionized air (or other ionized gas) is injected through a backwards facing slot placed slightly behind the stagnation point (Fig. 1). The body force that we consider to be acting in the ionized fluid is a streamwise electric field created by a series of plate electrodes transversal to the flow on the airfoil surface.

In addition to numerical calculations, we obtain scaling solutions and analytical design formulas. Here we simply report the main results. More detail may be obtained from a full report available as an electronic preprint [physics/9705020 at xxx.lanl.gov]. The conclusion is that, by itself or in conjunction with other methods, this technique may be useful in delaying the transition.

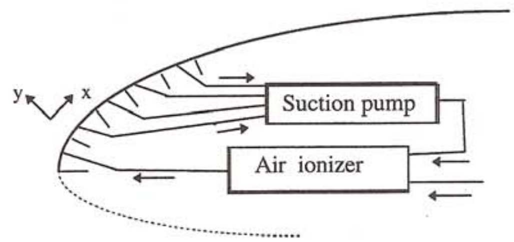

Fig. 1 Airfoil transversal cut showing ionized air injection, suction pump and plate electrodes

2 Ionized Boundary Layers With Electric Fields

The Boundary-Layer Equations. We use orthogonal curvilinear coordinates, with $\tilde{x}$ parallel to the surface along the flow, $\tilde{y}$ normal to the surface and assume that $\kappa\delta$ is small (κ being the curvature and δ the boundary layer thickness). Translational symmetry along the spanwise $\tilde{z}$-direction is assumed. Denote by $\tilde{u}$ and $\tilde{v}$ the components of the fluid velocity along the $\tilde{x}$ and $\tilde{y}$ directions. $\tilde{\rho}_m$ is the mass density, $\tilde{\sigma}_e$ the electric charge density and $\tilde{E}$ an applied electric field. The tilde denotes quantities in physical dimensions to be distinguished from the adimensional quantities defined below. Consider typical reference values L_r, δ_r, U_r, ρ_r, ν_r, σ_r, E_r for the airfoil width, the boundary layer thickness, the fluid velocity, the fluid mass density, the kinematic viscosity, the fluid charge density and the electric field. Then we define the adimensional quantities

$$t = \tilde{t}\frac{U_r}{L_r}, \quad x = \frac{\tilde{x}}{L_r}, \quad y = \frac{\tilde{y}}{\delta_r},$$

$$u = \frac{\tilde{u}}{U_r}, \quad v = \frac{\tilde{v}L_r}{U_r\delta_r}, \quad \rho_m = \frac{\tilde{\rho}_m}{\rho_r}$$

$$p = \frac{\tilde{p}}{\rho_r U_r^2}, \quad R_L = \frac{U_r L_r}{\nu_r},$$

$$\nu = \frac{\tilde{\nu}}{\nu_r}, \quad \sigma = \frac{\tilde{\sigma}}{\sigma_r}, \quad E = \frac{\tilde{E}}{E_r} \tag{1}$$

In general, $R_L \gg 1$. We neglect terms of order $1/R_L$ and δ_r^2/L_r^2 and also notice that the y-component of the electric field is suppressed by the factor δ_r/L_r.

Defining $u_e = u\,(y = \infty)$, a stream function ψ and a change of variables

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[1] Researcher, Grupo de Física-Matemática, Complexo II, Universidade de Lisboa, Av. Gama Pinto 2, 1699 Lisboa Codex, Portugal.

[2] Full Professor, Laboratório de Mecatrónica, DEEC, Instituto Superior Técnico, Av. Rovisco Pais, 1096 Lisboa Codex, Portugal.

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$$\eta = \left(\frac{u_e}{\nu\omega}\right)^{1/2} \frac{y}{\xi(x)}, \quad \psi = (u_e \nu \omega)^{1/2} \xi(x) f(x, \eta) \quad (2)$$

with $\omega = L_r^2/\delta_r^2 R_L = L_r \nu_r/\delta_r^2 U_r$, $\gamma = L_r \sigma_r E_r / U_r^2 \rho_r$, $u = \partial\psi/\partial y$ and $v = -(\partial\psi/\partial x)$, we obtain, for stationary solutions

$$\frac{\partial}{\partial\eta}\left(\frac{\partial^2 f}{\partial\eta^2}\right) + \xi \frac{\partial\xi}{\partial x} f \frac{\partial^2 f}{\partial\eta^2} + \frac{\xi^2}{u_e}\frac{\partial u_e}{\partial x} + \xi^2\left(\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial x} - \frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial x}\right) = -\frac{\gamma}{u_e^2 \rho_m}\xi^2(x)\sigma(x,\eta)E_x(x,\eta) \quad (3)$$

Scaling Solutions. With plate electrodes transversal to the fluid flow the mean electric field in the x-direction may be parametrized by

$$\bar{E}_x = g(x)\frac{l(x)}{l^2(x) + y^2} \quad (4)$$

$E_0 = g(x)/l(x)$ is the field at $y = 0$ and $l(x)$ is of the order of the electrode spacing. For thin boundary layers the field E_x may with good approximation be considered to be independent of y throughout the boundary layer thickness. Ionized gas is injected through a slot near the leading edge of the airfoil, being then carried along the airfoil surface by the flow. The steady-state charge distribution in the boundary layer is obtained from the continuity equation

$$\sigma(x, y) = \sigma_0(1 - d_1\psi(x, y))\theta(1 - d_1\psi(x, y))\theta(x - x_0) \quad (5)$$

where σ_0 is the injection intensity, d_1 the rate of depletion, ψ the stream function and x_0 the coordinate of the injection slot. The dynamically-dependent charge density profile may also be parametrized by the simpler formula

$$\sigma(x, y) = \sigma_0\left(1 - \frac{u}{u_e}\right) \quad (6)$$

A scaling solution is one for which f is a function of η only. Equation (3) becomes

$$(f'')' + \xi\dot{\xi} f f'' + \frac{\xi^2}{u_e}\frac{\partial u_e}{\partial x} = -\frac{\gamma}{u_e\rho_m}\xi^2(x)\sigma_0(1 - f')g(x)\frac{l(x)}{u_e l^2(x) + \omega\nu\xi^2(x)\eta^2} \quad (7)$$

with boundary conditions $f(0) = f'(0) = 0, f'(\infty) = 1$. We have denoted $f' \equiv \partial f/\partial\eta$ and $\dot{\xi} = \partial\xi/\partial x$. With the pressure approximately constant for length scales L of the order of the airfoil $\partial u_e/\partial x \approx 0$. The factorized nature of Eq. (7) implies that solutions exist only if

$$\xi^{-2}(x) = \frac{2}{c_1}\frac{\dot{\xi}(x)}{\xi(x)} = \frac{1}{c_3}\frac{g(x)}{\xi(x)} = c_4 l^{-2}(x) \quad (8)$$

c_1, c_3 and c_4 being constants. Therefore

$$\xi(x) = \sqrt{c_1 x + c_2}, \quad g(x) = \frac{c_3}{\xi(x)},$$

$$l(x) = \sqrt{c_4}\xi(x) \quad (9)$$

There are two interesting situations. The one with $c_1 \neq 0$ $c_2 = 0$ and the one with $c_1 = 0$ $c_2 \neq 0$. The first one corresponds to a boundary layer starting at $x = 0$ and growing with $x^{1/2}$ and the second to a constant thickness boundary layer. In the first case one chooses $c_2 = 0$ to obtain a boundary layer starting at $x = 0$. The scaling hypothesis requires then an electric field that is singular at $x = 0$, $y = 0$ ($E_x \sim x^{-1}$). In any case this electric field solution is not very interesting for our purposes because it leads to a boundary layer growth of $x^{1/2}$, as in the free force Blasius solution. In the second case the equation is

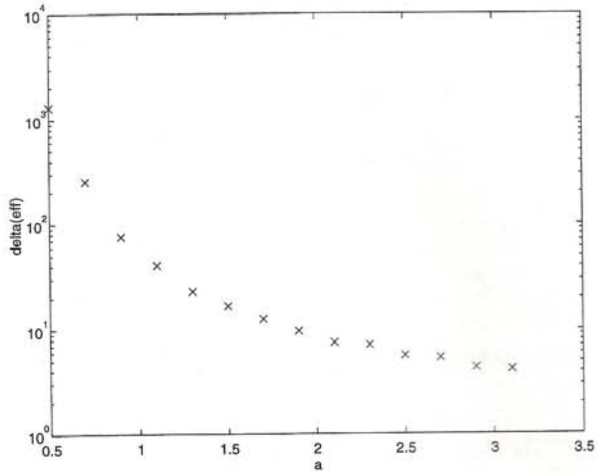

Fig. 2 Effective boundary layer thickness δ^* ($f'(\delta^*) = u/u_e = 0.95$) for the constant thickness scaling solution

$$f'''(\eta) + (1 - f'(\eta))\frac{a}{b^2 + \eta^2} = 0 \quad (10)$$

with $c_1 = 0$, $c_2 \neq 0$, $a = \gamma\sigma_0 c_3\sqrt{c_4}/u_e\beta\rho_m\nu\omega$, $b = \sqrt{u_e c_4/\nu\omega}$. With the replacement $\phi(\eta) = 1 - f'(\eta)$ and choosing $c_4 = \nu\omega/u_e$, which is a simple rescaling of ξ, Eq. (10) becomes the zero-eigenvalue problem for a Schrödinger equation in the potential $a/(1 + \eta^2)$. Using the WKB approximation we obtain

$$f'(\eta) = 1 - \frac{(1 + \eta^2)^{1/4}}{(\eta + \sqrt{1 + \eta^2})^{\sqrt{a}}} \quad (11)$$

which is a very good approximation to the exact solution for $a \geq 1$. Figure 2 shows the effective boundary layer thickness as a function of a. The effective boundary-layer thickness δ^* is defined here as the value of η at which the velocity u reaches 0.95 of its asymptotic value u_e. A very fast thinning of the boundary layer is obtained (several orders of magnitude) for a relatively short range of the a parameter. Figure 2 shows the variation of δ^* for small a. For large a (and small δ^*) one has the asymptotic formula

$$\delta^* \simeq \frac{2.9957}{\sqrt{a}}$$

If the longitudinal electric field E_x is assumed to be a constant (E_0) throughout the boundary-layer thickness, with the same charge profile, the solution is even simpler, namely

$$f'(\eta) = 1 - e^{-\eta\sqrt{h}} \quad (12)$$

with $\xi = \sqrt{c_2}$ and $h = \gamma c_2\sigma_0 E_0/\beta u_e^2\rho_m$.

For reference values of the physical quantities in Eqs. (1) we take

$$U_r = 100 \text{ m s}^{-1}, \quad L_r = 1 \text{ m},$$

$$\delta_r = 10^{-3} \text{ m}, \quad \rho_r = 1.2 \text{ Kg m}^{-3}$$

$$E_r = 500 \text{ V cm}^{-1}, \quad \sigma_r = 15\ \mu\text{C cm}^{-3},$$

$$\nu_r = 1.5 \times 10^{-5} \text{ m}^2 \text{ s}^{-1} \quad (13)$$

For these reference values, the adimensional constants ω and γ are $\omega = 0.15$ and $\gamma = 62.499$.

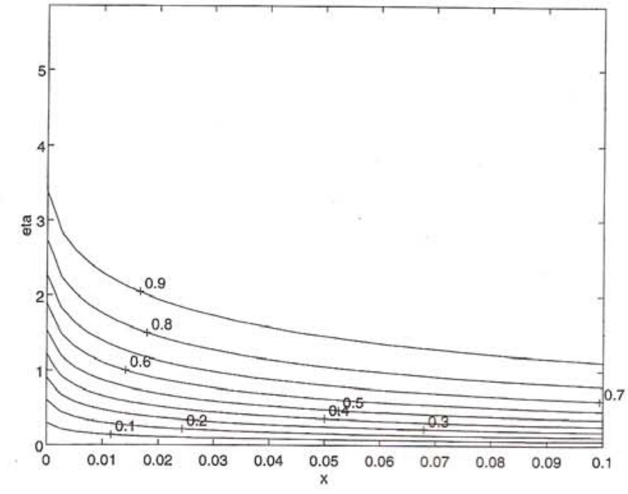

Fig. 3 Contour plot of $f'(x, \eta)$ for $S = 0.6$

Let y^* be the point at which $u/u_e = 0.95$. If stability of a laminar boundary layer cannot safely be guaranteed for local Reynold numbers $R_S = \tilde{u}_e \tilde{y}^*/\tilde{\nu}$ greater than about 10^3, we obtain for the classical force-free Blasius solution, and these reference parameters, that $\tilde{y}^* \simeq 0.15$ mm and, without control, the laminar region would be of the order of 1 cm, a tiny portion of a typical wing.

From the scaling solutions we may obtain an estimate of the controlling effect of a streamwise electric field on the boundary layer profile. The result is that a constant thickness boundary layer with local Reynolds number $R_S = 10^3$ is obtained for a charge density, in physical units, equal to $\tilde{\sigma}_0 = \sigma_0 \sigma_r = 14.36$ μC cm^{-3}.

The above estimate is obtained using the reference values for the kinematic variables. For other values we have the following designing formula (in normalized units)

$$\sigma_0 E_0 = 0.957 \frac{u_e^3 \rho_m}{10^{-6} R_S^2 \nu} \tag{14}$$

For design purposes, another important estimate concerns the total current I of the injected ionized gas, which determines the required ionization power. Integrating (6) over the boundary layer one obtains

$$\tilde{I} = 0.03675 L \sqrt{\frac{\sigma_0 \nu \rho_m u_e^3}{E_0}} \tag{15}$$

with the current in physical MKS units and all terms in the right-hand side being adimensional quantities normalized by the reference values.

3 Numerical Results

For the numerical solution of Eq. (3), with σ given by Eq. (6), we use an implicit finite-difference technique (Blottner, 1970; Davis, 1970; Hamilton et al., 1992). The electric field is parametrized as in Eq. (4), namely

$$E_x = E_0 \frac{\dfrac{u_e l^2}{\nu\omega}}{\dfrac{u_e l^2}{\nu\omega} + \xi^2(x)\eta^2}$$

with $u_e l^2/\omega = 666.66$ which corresponds to $l = 10$, $u_e = 1$ and $\nu\omega = 0.15$. For these parameters the electric field has only a small variation throughout the boundary layer region. For the scaling function we take $\xi(x) = \sqrt{x}$. Then all results depend only on the variable S

$$S = \frac{1}{62.499} \frac{\gamma}{u_e^2 \rho_m} \sigma_0 E_0$$

($S = 1$ when all quantities take the reference values). In Fig. 3 we show a contour plot of the numerical solution for $f'(x, \eta)$ ($= u/u_e$) when $S = 0.6$. From the x-dependence of the numerical solutions we may compute the effect of the electric field in extending the laminar part of the boundary layer. By defining, as in Section 2, the length of the laminar part as the x-coordinate corresponding to a local Reynolds number of 10^3 and denoting by x_0 ($u/u_e = 0.95$) the force-free value we have obtained for the ratio $R = x/x_0$ the results shown in Fig. 4. For $S = 0$ we obtain the Blasius solution and as we approach $S = 0.957$, corresponding to the scaling solution, the ratio diverges. The matching of the results in the force-free and scaling limits is a good check of the numerical algorithm. A clear indication of the results in Fig. 4 is that not much improvement is obtained unless one is able to obtain ionization charge densities of the order of the reference value σ_r.

4 Remarks

1 In this paper we have concentrated on controlling the profile of the boundary layer. The profile has a direct effect on the laminar or turbulent nature of the flow which, in a simplified manner, we estimated by a local Reynolds number $R_S = \tilde{u}_e \tilde{y}^*/\nu$) defined as a function of the effective thickness $\tilde{y}^*$. Another relevant aspect, of course, is the active control of the transition instabilities that can be achieved by electromagnetic body forces on the charged fluid. A simplified treatment of the Tollmien-Schlichting fluctuations leads to the conclusion that a space-time modulation of the electric field, with the appropriate phase, is equivalent to an effective viscous damping effect which delays the growth of the transition region instability. For this to be effective one needs to detect the phase of the wave instabilities by electromagnetic probes. Absolute synchronization of the feedback electric modulation is, however, not so critical as in acoustic noise cancelation, because here the objective is only to obtain an effective damping effect. The effective damping gives an intuitive understanding of why a feedback electric modulation might work. A more rigorous treatment requires the solution of an integro-differential eigenvalue problem. For details refer to Vilela Mendes (1997).

2 Because of the perturbation induced by the injection method, it seems advisable to use this method in conjunction with suction and passive control in the rear part of the airfoil.

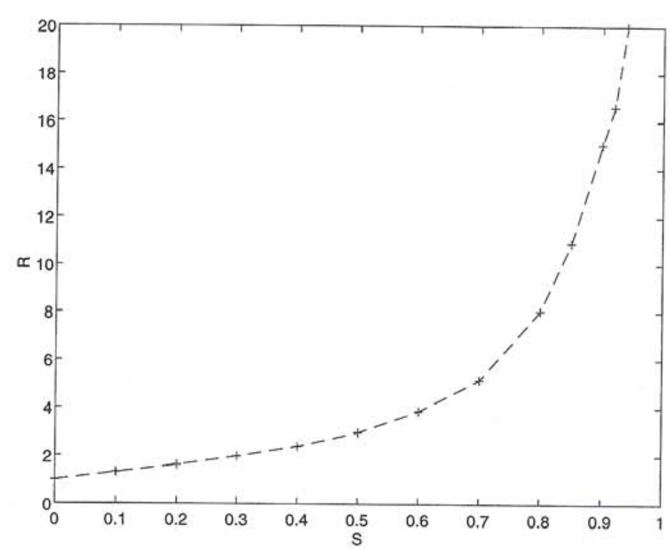

Fig. 4 Ratio of boundary layer laminar regions with and without electric field control

Even if a fully laminar boundary layer may never be completely achieved, any small improvement will become, in the long run, quite significant in terms of fuel consumption.

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